B. Test 1. Econ.Anal 13 Oct.21. NEPTUN: Name:

1. A. Compute the derivatives of the following functions!

1. $\sqrt[3]{\sin(3x)}$
2. $\sqrt[3]{x}\, tg(2x-1)$
3. $\frac{x^7}{sin(3x)}$

B. Let $f(x) = -x^2 - 2x$. Compute $\frac{f(5+\Delta x)-f(5)}{\Delta x}$! What is the limit of this fraction as $\Delta x \to 0$? What is $f'(5)$?

2. A. Study the monotonicity, convexity and local extremal values of the following function!
$f(x) = x^2 - x^4$.
Draw its graph!
B. Study the boundedness and convergence of the following sequence!
$\frac{3n+4}{5n+6}$.

3.A. Compute the limit of the following sequence! $a_n = \frac{2^{2n-88}}{3^{n+77}5^n}$.

B. Let $\phi(x) = 4x + 16$, $x_0 = 13$, $x_{n+1} = \phi(x_n)$. What are ϕ^{-1} and $\phi^n(1) = x_n$?

1. Find the fixed point x_f of ϕ !
2. Introduce $\Delta x = x - x_f$ and $\tilde{\phi}(\Delta x) = \phi(x_f + \Delta x) - x_f$. Calculate $\tilde{\phi}$ and $\tilde{\phi}^n$!
3. Compute x_n !

4. A. Let $\bar{v}_1 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$, $\bar{v}_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 12 \\ 8 \end{pmatrix} = \alpha\bar{v}_1 + \beta\bar{v}_2$. Compute $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$!

B. Let T be a 2×2 matrix formed by the transition probabilities of a two state (labeled by 1 and 2) stochastic system, where
$T(1 \leftarrow 1) = T_{11} = 0.5$, $T(2 \leftarrow 1) = T_{21} = 0.5$, $T(1 \leftarrow 2) = T_{12} = 0.5$, $T(2 \leftarrow 2) = T_{22} = 0.5$.

1. Find an eigenvector $\bar{v}_1$ corresponding to the eigenvalue $\lambda_1 = 1$! (This is the equilibrium state.)
2. Find the eigenvalue λ_2 of T corresponding to the eigenvector $\bar{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$!
3. Calculate α and β in $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \alpha\bar{v}_1 + \beta\bar{v}_2$!
4. Calculate $T(\alpha\bar{v}_1 + \beta\bar{v}_2)$, $T^2(\alpha\bar{v}_1 + \beta\bar{v}_2)$, etc.