

1. A. Compute the derivatives of the following functions!

1.  $e^x \cos(2x - 1)$

2.  $e^7 \ln(2x - 1)$

3.  $\frac{\ln(2x)}{\ln(x)}$

B. What is the prediction of the linear approximation of the function  $f(x)$  at  $x = x_0$  for the value of  $f(x_0 + \Delta x)$  ?

$f(x) = \ln x$ ,  $x_0 = e$ ,  $\Delta x = 0.1$ .

2. A. Study the monotonicity, convexity and local extremal values of the following function!

$f(x) = 2x^3 - 3x^2$ .

Draw its graph!

B. Study the monotonicity of the following sequence!

$\frac{3n+4}{5n+6}$ .

3. A. Compute the limit of the following sequence!  $a_n = \left(1 + \frac{4}{3n}\right)^{3n-7}$ .

B. Let  $\phi(x) = 3x - 9$ ,  $x_0 = 13$ ,  $x_{n+1} = \phi(x_n)$ . What are  $\phi^{-1}$  and  $\phi^n(1) = x_n$  ?

1. Find the fixed point  $x_f$  of  $\phi$  !

2. Introduce  $\Delta x = x - x_f$  and  $\tilde{\phi}(\Delta x) = \phi(x_f + \Delta x) - x_f$ . Calculate  $\tilde{\phi}$  and  $\tilde{\phi}^n$  !

3. Compute  $x_n$  !

4. A. Let  $\phi\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} -y \\ x + 2y \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix}$ ,  $\phi\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} 2x + 4y \\ +y \end{pmatrix} = B\begin{pmatrix} x \\ y \end{pmatrix}$ . Calculate  $A$  and  $B$  ! Let  $\phi\left(\psi\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)\right) = C\begin{pmatrix} x \\ y \end{pmatrix}$ . Compute  $C$  !

B. Let  $\phi\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} 2y \\ 7x + y \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix}$ . Calculate the  $A^{-1}$  matrix of the inverse  $\phi^{-1}$  mapping!

1. Calculate  $\det(A)$  ! Does  $A^{-1}$  exist? Why?

2. Write down the matrix equation that defines  $A^{-1}$  !

3. Write down and solve the corresponding linear system of scalar equations!

4. Use  $A^{-1}$  to find the solution of the system of equations

$$2y = 12$$

$$7x + 1y = 13.$$