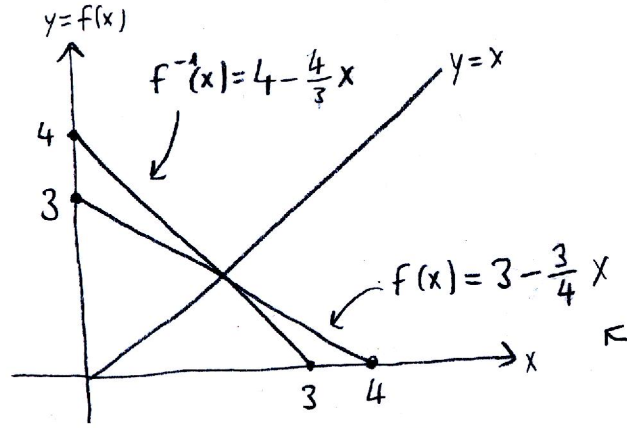
Econ. Math. Test. 1. Oct. 22

5 points

①



ⓐ $f(x)=3-\frac{3}{4}x$

ⓑ Slope of f: $-3/4$

ⓒ $y=3-\frac{3}{4}x \rightarrow \frac{3}{4}x=3-y \rightarrow x=\frac{4}{3}(3-y)=4-\frac{4}{3}y$

ⓓ $f^{-1}(x)=4-\frac{4}{3}x$

← ⓔ

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5 points

② $f(x)=4\cdot 2^{x+1}$

ⓐ Inverse: $y=4\cdot 2^{x+1}=2^{x+3}$

$$\frac{y}{4}=2^{x+1}$$

$$\log_2\frac{y}{4}=x+1$$

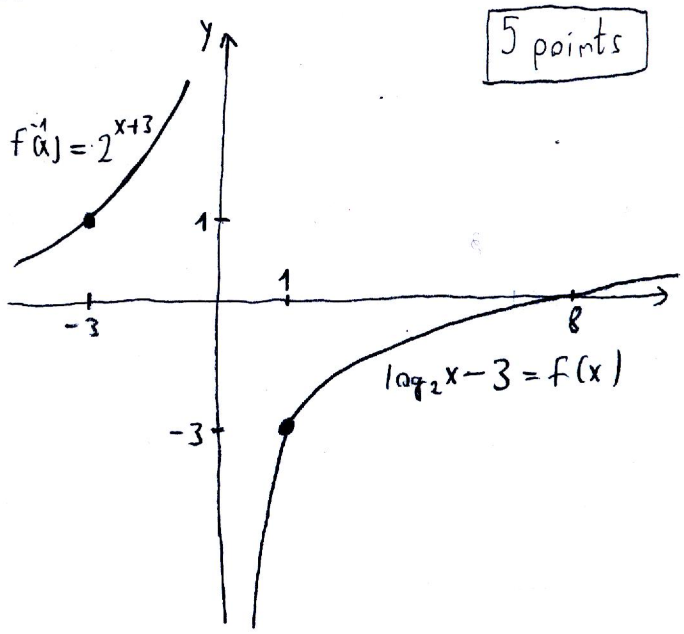
$$x=\log_2\frac{y}{4}-1$$

$$f^{-1}(y)=\log_2\frac{y}{4}-1$$

$$f^{-1}(x)=\log_2\frac{x}{4}-1$$

$$=\log_2 x-\log_2 4-1$$

$$=\log_2 x-3$$



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5 points

③ $x_0=0$, $x_{n+1}=f(x_n)=3x_n+2$

ⓐ Fixed point: $x_{fix}=3x_{fix}+2 \longrightarrow x_{fix}=-1$

ⓑ $x_n=3^n(0-(-1))+(-1)=3^n-1$

ⓒ $x_3=33=3\cdot x_2+2 \longrightarrow x_2=f^{-1}(x_3)=f^{-1}(33)=\frac{31}{3}$

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④ ⓐ $a_n=\frac{1-2n}{3n-2}$, $a_{n+1}-a_n=\frac{1-2(n+1)}{3(n+1)-2}-\frac{1-2n}{3n-2}=\frac{(-2n-1)(3n-2)-(1-2n)(3n+1)}{(3n+1)(3n-2)}=$

$=\frac{1}{(3n+1)(3n-2)}>0$ stricty increasing

ⓑ ⓒ limits, boundedness

$\underbrace{\frac{1-2n}{3n-2}}_{\text{bounded}}\longrightarrow-\frac{2}{3}$, $\underbrace{(-1)^n\frac{1-2n}{3n-2}}_{\text{bounded}}$ divergent, $\underbrace{(-1)^n\frac{3n^2-n+3}{n+5n^2+1}}_{\text{bounded}}$ divergent

$\lim_{n\to\infty}\left(1-\frac{3}{4n}\right)^{-3n+77}=\lim_{n\to\infty}\left[\left(1+\frac{-3/4}{n}\right)^n\right]^{-3}\cdot\left(1-\frac{3}{4n}\right)^{77}=\left[e^{-3/4}\right]^{-3}\cdot 1^{77}=e^{9/4}\rightarrow$ bounded

$\left(\frac{1}{3}-\frac{3}{4n}\right)^{-3n+77}\approx\left(\frac{1}{3}\right)^{-3n}\cdot\left(\frac{1}{3}\right)^{77}=3^{3n}\cdot\left(\frac{1}{3}\right)^{77}\rightarrow\infty$, unbounded

10 points

5) $f(x) = x - 3x^2 + 2, \quad x_0 = 3$

$$\frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} = \frac{[(3+\Delta x) - 3\cdot(3+\Delta x)^2 + 2] - [3 - 3\cdot 3^2 + 2]}{\Delta x}$$

$$= \frac{[(3+\Delta x) - (27 + 18\Delta x + 3\Delta x^2) + 2] - [3 - 27 + 2]}{\Delta x}$$

$$= \frac{-17\Delta x - 3\Delta x^2}{\Delta x} = -17 - 3\Delta x$$

$\lim\limits_{\Delta x \to 0} -17 - 3\Delta x = -17 = f'(3)$, indeed $f'(x) = 1 - 3\cdot 2x$, $f'(3) = 1 - 3\cdot 2\cdot 3 = -17$

linear approximation:

$f(3) = 3 - 3\cdot 3^2 + 2 = -22$

$f'(3) = -17$

$f(3+0.01) \approx -22 + (-17)\cdot 0.01$

$\boxed{\text{6 points}}$

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6) $$\left[\frac{1}{(3x)^2} + \frac{1}{\sqrt[3]{x}} + \frac{1}{\sqrt[3]{5x}}\right]' = \left[(3x)^{-2} + x^{-1/3} + (5x)^{-1/3}\right]' =$$

$$= -2\cdot(3x)^{-3}\cdot 3 + \left(-\tfrac{1}{3}\right) x^{-4/3} + \left(-\tfrac{1}{3}\right)(5x)^{-4/3}\cdot 5$$

$$\left[\frac{\cos(4x+1)}{x^3}\right]' = \frac{[-\sin(4x+1)\cdot 4]\cdot x^3 - \cos(4x+1)\cdot[3x^2]}{(x^3)^2}$$

$$\left[\ln\left(\ln(4x+1)\right)^2\right]' = \frac{1}{(\ln(4x+1))^2} \cdot \left[(\ln(4x+1))^2\right]' =$$

$$= \frac{1}{(\ln(4x+1))^2} \cdot 2\cdot \ln(4x+1) \cdot [\ln(4x+1)]' =$$

$$= \frac{1}{(\ln(4x+1))^2} \cdot 2\cdot \ln(4x+1) \cdot \frac{1}{4x+1} \cdot 4$$

$\boxed{\text{10 points}}$

7 **10 points**

$f(x) = (x+1)\,e^{x+1}$

$f'(x) = 1\cdot e^{x+1} + (x+1)\cdot e^{x+1} = (x+2)\,e^{x+1}$

$f''(x) = 1\cdot e^{x+1} + (x+2)\cdot e^{x+1} = (x+3)\,e^{x+1}$

$\lim\limits_{x\to\infty} (x+1)\,e^{x+1} = \infty\cdot\infty = \infty$

$\lim\limits_{x\to\infty} (x+1)\,e^{x+1} = \infty\cdot 0$ which is 0 here, as numbers like $(-1000)\,e^{-1000}$ are very small.
$= 0$

Extremal value:

$f'(x) = 0$, $(x+2)\,e^{x+1} = 0 \longrightarrow x+2=0 \longrightarrow x=-2$

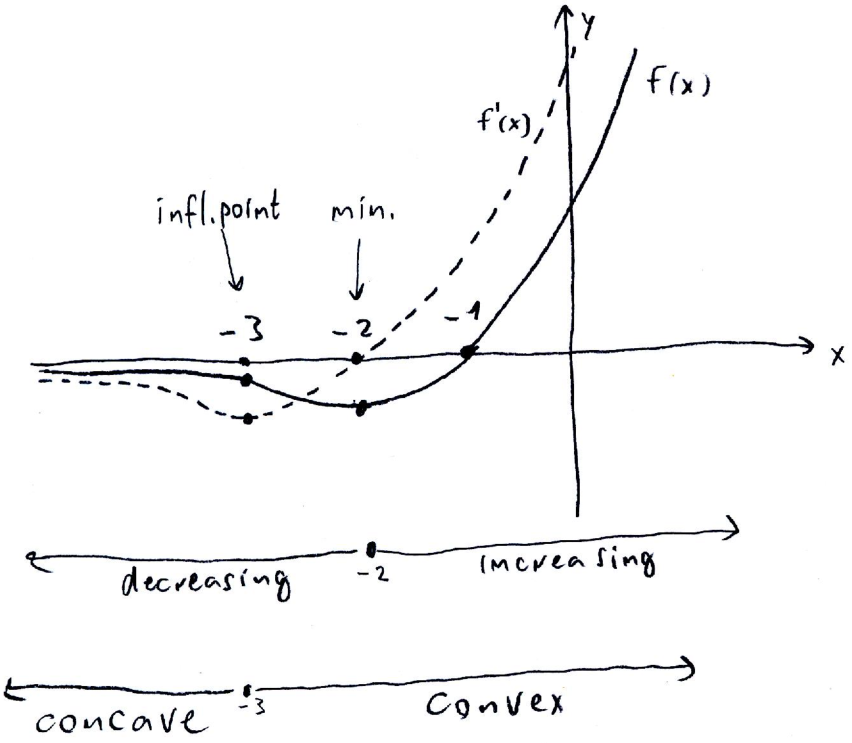
$f''(-2) = (-2+3)\,e^{-2+1} = e^{-1} > 0 \longrightarrow$ MINIMUM

Convexity:

$f''(x)=0 \quad (x+3)\,e^{x+1} = 0 \longrightarrow x=-3$

$x\in(-\infty,-3) \longrightarrow f''<0$ concave

$x\in(-3,\infty) \longrightarrow f''>0$ convex



Total: 51 points
pass: 18–51
fail: 0–17