Econ.Math.Test.1.Exercises

The problems of the first test will be quite similar to the problems of 15.test1, 15.test1.solutions. Expect problems similar to these:

1. Let $f(x) = y = 3x - 6$.
   - Where are the intersections of this line with the x and y axes?
   - What is the slope of this function?
   - Express x with y !
   - What is the $f^{-1}(x)$ inverse of the function $f(x)$?
   - Plot $f(x)$ and $f^{-1}(x)$ together!
2.
   - Write down the $y = f(x)$ equation of the straight line going through the points $(0, 6)$ and $(3, 0)$.
   - What is the slope of $f(x)$?
   - Express x with y !
   - What is the $f^{-1}(x)$ inverse of the function $f(x)$?
   - Plot $f(x)$ and $f^{-1}(x)$ together!
3. Let $f(x) = 0.5 \cdot 2^x$.
   - Compute $f^{-1}(x)$!
   - Plot $f(x)$ and $f^{-1}(x)$ together!
4. Let $f(x) = \lg(2x) + 3$.
   - Compute $f^{-1}(x)$!
   - Plot $f(x)$ and $f^{-1}(x)$ together!
5. Let $x_0 = 13,\ x_{n+1} = f(x_n) = 0.9x_n + 2$.
   - If $f(x_{fix}) = x_{fix}$, then how much is x_{fix} ?
   - How much is x_n ?
   - If $x_3 = 33$, how much is x_2 ?
6. Study the following sequences with regard to:
   - monotonicity,
   - boundedness,
   - limit, convergence.

$$\frac{2n+1}{3n-2}, \quad (-1)^n \frac{2n+1}{3n-2}, \quad \frac{1}{3n-2}, \quad (-1)^n \frac{1}{3n-2},$$
$$\frac{3n^2-n+3}{n+5n^2+1}, \quad \frac{3n^3-n+3}{n+5n^2+1}, \quad \frac{3n^2-n+3}{n+5n^3+1},$$
$$(1+3/(4n))^n, \quad (1+3/(4n))^{3n-77}, \quad (3+3/(4n))^n, \quad (1/3+3/(4n))^{3n-77},$$
$$(-1)^n (1+3/(4n))^n, \quad (-1)^n (1+3/(4n))^{3n-77}, \quad (-1)^n (3+3/(4n))^n, \quad (-1)^n (1/3+3/(4n))^{3n-77}.$$

7. Let $f(x) = 3x^2 - 5x + 2$, $x_0 = 2$.
   - Compute
   $$\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$
   - What is the limit of the previous expression as $\Delta x \to 0$?
   - What is the prediction of the linear approximation of $f(x)$ around x_0 for the value of $f(x_0 + 0.01)$?
   - Repeat this exercise for $f(x) =$
   $$0, \quad 1, \quad x, \quad x^2, \quad x^3.$$
8. Compute the derivatives of the following functions:

$$-1, \quad 1/x, \quad 1/\sqrt[2]{x}, \quad 1/\sqrt[3]{5x}, \quad -1 + 1/x + 1/\sqrt[2]{x} + 1/\sqrt[3]{5x},$$
$$x^{-2}\cos(4x+1), \quad \frac{\cos(4x+1)}{x^2}, \quad x^3\cos(4x+1),$$
$$\cos(4x^2+1), \quad (\cos(4x+1))^2, \quad \ln\left((\cos(4x+1))^2\right).$$

9. Study the monotonicity, convexity and the local extremal values of the following functions! Find their limits as $x \to \pm\infty$! Draw also the graphs of $f(x)$ and $f'(x)$ in the same coordinate system!

$$1, \quad x, \quad x^2, \quad x^3, \quad 1-x,$$
$$x^3-3x, \quad x^3+3x, \quad x^3-x^2, \quad x^3+3x^2, \quad x^4-x^2,$$
$$e^x x, \quad -e^{-x}x, \quad e^{2x+1}x, \quad e^x(2x+1).$$