

I.

A. Compute  $f'(x)$  and  $f'_y(x, y)$  !

- $f(x) = \sqrt[4]{(3x)^3} - \frac{x}{x^4} + \ln(4 - 3x)$
- $f(x) = \ln x e^{(-x-1)}$
- $f(x) = \frac{\tan(2x)}{x+(3x)^2}$
- $f(x, y) = \frac{y+x}{2y-3x}$

B. Compute the following definite integrals!

- $\int_0^1 e^{3x} \cdot (2x + 3) dx$
- $\int_1^2 e^{3x^2} \cdot 2x dx$

2.

- There are 3 black and 5 white balls in a box. Suppose that we DO put back the balls after the drawings. What is the chance of drawing firstly 6 white and then 2 black balls? What is the chance of drawing 5 white and then 3 black balls if the order is irrelevant?
- Suppose that we roll a dice. Six numbers (from 1 to 6) can appear face up with equal chances. So our sample space is:  $\Omega = \{1, 2, 3, 4, 5, 6\}$ . Define the events  $E$  and  $F$  as follows:  $E = \{2, 4, 6\}$ ,  $F = \{3, 4, 5, 6\}$ . Are  $E$  and  $F$  independent? Prove your answer!  
Now roll the dice twice. What is the chance that both  $E$  and  $F$  will happen once?
- There were two boxes, one containing 15 silver and 5 gold coins, while the other had 18 gold and only two silver coins. I was allowed to draw a random coin from the box of my choice. I got a silver coin, so I took the other box. What was my chance that I picked the box almost full of gold?

3.

A. Let  $f(x) = 7 - x^2$ . Compute  $\frac{f(3+\Delta x_n) - f(3)}{\Delta x_n}$  ! What is the limit of this fraction as  $n \rightarrow \infty$  if  $\Delta x_n = 1/n^2$  ? How much is  $f'(3)$  ?

B. Study the following sequences! (convergence, limit, monotonicity, upper and lower bounds)

- $\frac{3-n}{2n+1}$ ,
- $\frac{(-1)^n}{2n^n+1}$ .

C. Compute the following limits!

- $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^2 - 1}$ ,
- $\lim_{x \rightarrow \infty} \frac{x}{2^x}$ .

4. Let  $f(x) = x^2 - x^3$ . Plot it, determine its domain, range and roots, find its local minima and maxima, find the intervals where it is increasing or decreasing, convex or concave.