Math.Econ.Anal.Test.1

1. (3+4+3 points)
   A) $\bar{m} = (-2,2)^T$, $\bar{p} = (2,2)^T$. Solve $\alpha\bar{m} + \beta\bar{p} = (10,4)^T$ for α and β !

   B) Let $a_n = \frac{3n+2}{5n+4}$. Is a_n increasing or decreasing? (Prove it!)

   Is a_n convergent as $n \to \infty$?

   If the answer is yes, what is the limit of a_n ?

   C) Compute $\lim_{n\to\infty} \left(1 - \frac{6}{5n}\right)^{5n-2}$!

2. (2+4+4 points)
   A) Let $\bar{r}_0 = (1,1,1)^T$ and $\bar{n} = (2,2,2)^T$. Find an equation of the plane that contains $\bar{r}_0$ and has normal vector $\bar{n}$!

   Express the third coordinate $z(x,y)$ of the point (x,y,z) of the plane with x and y !

   B) Let $f(x) = 3x + 4$. If $a_0 = 13$ and $a_{n+1} = f(a_n) = 3a_n + 4$, then how much is a_n ?

   B) How much is $|(3,4,5)^T|$? How much is $(3,1,4,5)^T (1,3,4,5)^T$?

3. (5×2 points)

   (a) Compute $(x^3 \cos(4x))'$!

   (b) Compute $(\cos(\sin(4x)))'$!

   (c) Compute $((\sin(-2x))^2)'$!

   (d) Compute $((-x)^3 + \sin(4x) - \sqrt[3]{(3x)^5})'$!

   B) Let $f(x) = e^{-x}$, $x_0 = 1$. What is the prediction of the linear approximation of f around x_0 for the value of $f(x_0 + \Delta x)$?

4. (3+2+5 points)
   A) Let $f(x) = 2^x$, $x_0 = 1$. Compute $\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$!

   B) Compute $\lim_{n\to\infty} \left(0.4 - \frac{6}{5n}\right)^{5n-2}$!

   C) Study the monotonicity, convexity and the local extremal values of the function $f(x) = x - x^2$!
   Draw the graphs of f and f' in the same coordinate system!

5. Compute
$$\int \frac{1}{x^2} + \sqrt[7]{(3x)^5} + \sin(7x - 1)\, dx, \quad \int e^{3x} x\, dx, \quad \int e^{3x^2} x\, dx$$

6. Solve the following differential equations:

   - $y'(x) = 2x - 77$. What are the general solutions?
   - $y'(x) = 2x - 77$, $y(3) = 5$. What are is the particular solution?