A • Legyen $f(x) = \cos(3x)$! Szamold ki f harmadrendu Taylor-polinomjat az $x = 0$ pont korul!

B • Keresd meg az $f(x) = x^2 - 2x^3$ fuggveny szelsoertekeit es hatarozd meg azok tipusat!

A (2p)

$f(x) = \cos(3x)$

$f'(x) = -\sin(3x) \cdot 3$

$f''(x) = -\cos(3x) \cdot 3^2$

$f'''(x) = \sin(3x) \cdot 3^3$

$x = 0$ (1p)

$f(0) = \cos(3 \cdot 0) = 1$

$f'(0) = -0 \cdot 3 = 0$

$f''(0) = -1 \cdot 3^2 = -9$

$f'''(0) = 0 \cdot 3^3 = 0$

$$\cos(3x) \approx 1 + 0x + \frac{(-9)}{2!} x^2 + \frac{0}{3!} x^3 = 1 - \frac{9}{2} x^2$$ (2p)

B

$f(x) = x^2 - 2x^3$

$f'(x) = 2x - 6x^2$ (1p)

$f''(x) = 2 - 12x$

szélsőértékek lehetséges helyei: $f'(x) = 0$

$$0 = 2x - 6x^2 = 2x(1 - 3x)$$

(1p) $x_1 = 0$

$f''(x_1) = 2 - 12 \cdot 0 = 2 > 0$

(1p) MIN

$x_2 = \frac{1}{3}$ (1p)

$f''(x_2) = 2 - 12 \cdot \frac{1}{3} = -2 < 0$

MAX (1p)

A • Ird fel az $f(x) = x^3 + x^2$ fuggvenynek az $x = 2$ pontba huzott erintojenek az $y(x)$ egyenletet!

B • Legyen $f(x) = x^3 + x^2$. Mennyi $\frac{f(2+\Delta x)-f(2)}{\Delta x}$?

C • Mennyi az elozo kifejezes limesze ahogy $\Delta x \to 0$?

D • Legyen $f(x) = 1/x^2$, $x_0 = 3$. Ird fel $f(x)$-nel az $y(x)$ linearis kozeliteset az x_0 pont korul!

E • Adj meg a kozelites $|f(x_0 + \Delta x) - y(x_0 + \Delta x)|$ hibajara minel jobb becslest Δx fuggvenyeben (azon felteves mellett, hogy $\Delta x \in [0, 0.1]$) !

2p A $f(2) = 2^3+2^2 = 12$, $f'(x) = 3x^2+2x$, $f'(2) = 3\cdot 2^2 + 2\cdot 2 = 16$

$y(2+\Delta x) = 12 + 16\Delta x$, $y(x) = 12 + 16(x-2) = 16x - 20$

2p B $$\frac{f(2+\Delta x)-f(2)}{\Delta x} = \frac{[(2+\Delta x)^3 + (2+\Delta x)^2] - [2^3+2^2]}{\Delta x} =$$

$$= \frac{[(2^3 + 3\cdot 2^2\cdot \Delta x + 3\cdot 2\Delta x^2 + \Delta x^3) + (2^2 + 2\cdot 2\cdot \Delta x + \Delta x^2)] - [2^3+2^2]}{\Delta x} =$$

$$= \frac{16\Delta x + 7\Delta x^2 + \Delta x^3}{\Delta x} = 16 + 7\Delta x + \Delta x^2$$

1p C $$\lim_{\Delta x \to 0} 16 + 7\Delta x + \Delta x^2 = 16$$

2p D $f(3) = \frac{1}{3^2} = \frac{1}{9}$, $f'(x) = -2\cdot x^{-3}$, $f'(3) = -\frac{2}{3^3} = -\frac{2}{27}$

$y(3+\Delta x) = \frac{1}{9} - \frac{2}{27}\Delta x$, $y(x) = \frac{1}{9} - \frac{2}{27}(x-3) = -\frac{2}{27}x - \frac{1}{9}$

E $f''(x) = -2\cdot(-3)x^{-4} = \frac{6}{x^4}$

1p $$|hiba(\Delta x)| \leq \frac{1}{2}\Delta x^2 \cdot \max_{z\in[x_0, x_0+\Delta x]} |f''(z)|$$

$$\max_{z\in[3,3+\Delta x]} \left|\frac{6}{z^4}\right| = \frac{6}{3^4} = \frac{2}{27} \qquad (\Delta x \in [0, 0.1])$$

2p $$|hiba(\Delta x)| \leq \frac{1}{2}\Delta x^2 \cdot \frac{2}{27} = \frac{\Delta x^2}{27}$$

3. (((2+1+2)+(2+3) pont)

- Legyen $f(x) = x^3 - 5x$.
  - Mennyi $\frac{f(1+\Delta x)-f(1)}{\Delta x}$? (Segitseg: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$)

$$= \frac{[(1+\Delta x)^3 - 5(1+\Delta x)] - [1^3 - 5\cdot 1]}{\Delta x} =$$

$$= \frac{[(1+3\Delta x + 3\Delta x^2 + \Delta x^3) - (5+5\Delta x)] - [1^3 - 5]}{\Delta x} = \frac{-2\Delta x + 3\Delta x^2 + \Delta x^3}{\Delta x} =$$

$$= -2 + 3\Delta x + \Delta x^2$$

  - Mennyi az elozo kifejezes limesze ahogy $\Delta x \to 0$?

$$\lim_{\Delta x \to 0} -2 + 3\Delta x + \Delta x^2 = -2$$

  - Ird fel f erintojenek az $y(x)$ egyenletet az $x_0 = 1$ pontban!

$$f'(1) = -2 \ , \ f(1) = 1^3 - 5\cdot 1 = -4$$

$$y(x) = f(1) + f'(1)\cdot(x-1) = -4 - 2(x-1) =$$

$$= -2x - 2$$

- Mennyi $\lim_{x\to 0} \frac{e^{-5x}-1}{tg(7x)}$?

$$\lim_{x\to 0} \frac{e^{-5x}-1}{tg(7x)} = \frac{0}{0} = \lim_{x\to 0} \frac{e^{-5x}\cdot(-5)}{\frac{1}{\cos^2(7x)}\cdot 7} = \frac{e^{-5\cdot 0}\cdot(-5)}{\frac{1}{\cos^2(7\cdot 0)}\cdot 7} = -\frac{5}{7}$$

- Mennyi $\lim_{x\to\infty} \frac{x}{\ln(x)}$?

$$\lim_{x\to\infty} \frac{x}{\ln x} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{1}{1/x} = \lim_{x\to\infty} x = \infty$$

3. (2+3+2+3 pont) Legyen $f(x) = x^2 - x^3$. $= x^2(1-x) = -1 \cdot x^2 \cdot (x-1)$

- Mennyi f, f' f''?

$f'(x) = 2x - 3x^2$

$f''(x) = 2 - 6x$

- Hol vannak f szelsoertekei es milyen tipusuak?

$f'(x) = 2x - 3x^2 = x(2-3x) = 0 \longrightarrow x_1 = 0 \qquad x_2 = \frac{2}{3}$

$f''(x_1) = f''(0) = \qquad f''\left(\frac{2}{3}\right) =$

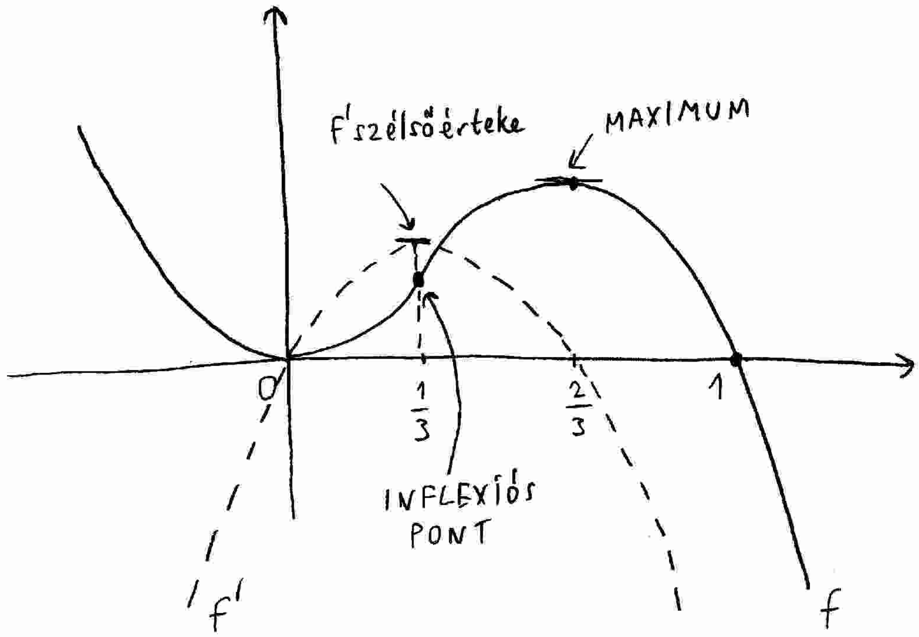
$= 2 - 6 \cdot 0 = 2 > 0 \qquad = 2 - 6 \cdot \frac{2}{3} = -2 < 0$

(lokális) MINIMUM MAXIMUM

- Hol van f inflexios pontja?

$f''(x_{infl}) = 2 - 6x = 0 \longrightarrow x_{infl} = \frac{1}{3}$

- Rajzold le f-t es f'-t (utobbit szaggatott vonallal) ugyanarra az abrara!

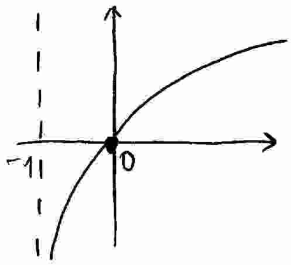


C. Zh2, Matematika I, 2012.december. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.

- Rajzold le az alabbi fuggvenyt! $f(x) = \ln(x+1)$



- Szamold ki az alabbi fuggveny inverzet! $f(x) = e^{x-3}$

$y = e^{x-3}$

$\ln y = \ln(e^{x-3}) = x - 3$

$x = \ln y + 3$

$f^{-1}(y) = \ln y + 3$

$f^{-1}(x) = \ln x + 3$

- Szamold ki a kovetkezo sorozat hatarerteket ahogy $n \to \infty$! $a_n = \left(2 - \frac{2}{5n}\right)^n$

$$\lim_{n\to\infty} \left(2 - \frac{2}{5n}\right)^n = \lim_{n\to\infty} 2^n = \infty$$

- Szamold ki a kovetkezo fuggveny x szerinti derivaltjat! $f(x) = \cos(-2x - 9)$

$$\left[\cos(-2x+9)\right]' = -\sin(-2x+9) \cdot (-2)$$

- Legyen $f(x) = 4x + x^2$. Melyik x_{sz} pontban nulla f derivaltja? Mennyi $f''(x_{sz})$? Milyen szelsoerteke van f-nek az x_{sz} pontban?

$f'(x) = 4 + 2x \longrightarrow 4 + 2x_{sz} = 0 \longrightarrow x_{sz} = -2$

$f''(x) = 2$

$f''(x_{sz}) = f''(-2) = 2 > 0$

$\downarrow$

MINIMUM

2. (5 × 2 pont) Szamold ki a kovetkezo fuggvenyek derivaltjait!

- $tg(3x)\ln(-3x)$

$$[tg(3x)\cdot\ln(-3x)]' = [tg(3x)]'\cdot\ln(-3x) + tg(3x)\cdot[\ln(-3x)]' =$$

$$= \frac{1}{\cos^2(3x)}\cdot 3\cdot\ln(-3x) + tg(3x)\cdot\frac{1}{-3x}\cdot(-3)$$

- $\sin(\sqrt{x+1})$

$$[\sin(x+1)^{1/2}]' = \sin'((x+1)^{1/2})\cdot[(x+1)^{1/2}]' =$$

$$= \cos((x+1)^{1/2})\cdot\frac{1}{2}(x+1)^{-1/2}$$

- $\left(\sqrt[2]{8x} + \frac{1}{(3x)^4} + tg(x) + ctg(4x)\right)' = \frac{1}{2}(8x)^{-1/2}\cdot 8 + (-4)(3x)^{-5}\cdot 3 + \frac{1}{\cos^2 x} - \frac{1}{\sin^2(4x)}\cdot 4$

$\sqrt[2]{8x} \to (8x)^{1/2}$

$\frac{1}{(3x)^4} \to (3x)^{-4}$

- $\frac{\sqrt{4x-2}}{\sin(x-1)}$

$$\left[\frac{(4x-2)^{1/2}}{\sin(x-1)}\right]' = \frac{[(4x-2)^{1/2}]'\cdot\sin(x-1) - (4x-2)^{1/2}\cdot[\sin(x-1)]'}{(\sin(x-1))^2}$$

$$= \frac{\frac{1}{2}(4x-2)^{-1/2}\cdot\sin(x-1) - (4x-2)^{1/2}\cdot\cos(x-1)}{(\sin(x-1))^2}$$

- $2^{3x}e^{-x+2}$

$$[2^{3x}\cdot e^{-x+2}]' = [2^{3x}]'\cdot e^{-x+2} + 2^{3x}\cdot[e^{-x+2}]' =$$

$$= [\ln 2\cdot 2^{3x}\cdot 3]\cdot e^{-x+2} + 2^{3x}\cdot e^{-x+2}\cdot(-1)$$

vagy

$$= [e^{\ln 2\cdot(3x) + (-x+2)}]' = [e^{\ln 2\cdot(3x) - x+2}]' =$$

$$= e^{\ln 2\cdot(3x) - x+2}\cdot(\ln 2\cdot 3 - 1)$$

3. (5+5 pont)

$$f(0) = 2 + e^{0+1} = 2 + e$$

- Legyen $f(x) = 2 + e^{x+1}$!. Szamold ki f harmadrendu Taylor-polinomjat az $x = 0$ pont korul!

$f'(x) = e^{x+1}$ $\quad f'(0) = e$

$f''(x) = e^{x+1}$ $\quad f''(0) = e$

$f'''(x) = e^{x+1}$ $\quad f'''(0) = e$

$$f(x) \approx T_3(x) = (2+e) + e \cdot x + \frac{e}{2!}x^2 + \frac{e}{3!}x^3 =$$

$$= 2 + e + ex + \frac{e}{2}x^2 + \frac{e}{6}x^3$$

- Keresd meg az $f(x) = 2x^2 - x^4$ fuggveny szelsoertekeit es hatarozd meg azok tipusat!

$$f'(x) = 4x - 4x^3 = 4x(1-x^2) = 4x(1-x)(1+x)$$

$$4x(1-x)(1+x) = 0 \longrightarrow$$

$$\longrightarrow x_1 = 0 \qquad x_2 = 1 \qquad x_3 = -1$$

$$f''(x) = 4 - 12x^2$$

$f''(x_1) = f''(0) = 4 - 12 \cdot 0^2 = 4$

$4 > 0$

MINIMUM

$f''(x_2) = f''(1) = 4 - 12 \cdot 1^2 = -8$

$-8 < 0$

MAXIMUM

$f''(x_3) = f''(-1) = 4 - 12 \cdot (-1)^2 = -8$

$-8 < 0$

MAXIMUM

2. (5 × 2 pont) Szamold ki a kovetkezo fuggvenyek derivaltjait!

- $\frac{2+x^3}{3+x^2}$

$$\left(\frac{2+x^3}{3+x^2}\right)' = \frac{(2+x^3)'\cdot(3+x^2) - (2+x^3)\cdot(3+x^2)'}{(3+x^2)^2} =$$

$$= \frac{3x^2\cdot(3+x^2) - (2+x^3)\cdot 2x}{(3+x^2)^2}$$

- $\left(\sqrt[3]{x^7} + \frac{1}{x^3} + \sin(3x) + \ln(2x)\right)' =$

$$\underset{x^{7/3}}{\uparrow} \quad \underset{x^{-3}}{\uparrow}$$

$$= \frac{7}{3}x^{4/3} + (-3)x^{-4} + \cos(3x)\cdot 3 + \frac{1}{2x}\cdot 2$$

- $\frac{\ln(4x)}{\frac{1}{x}+1}$

$$\left[\frac{\ln(4x)}{\frac{1}{x}+1}\right]' = \frac{[\ln(4x)]'\cdot\left(\frac{1}{x}+1\right) - \ln(4x)\left[\frac{1}{x}+1\right]'}{\left(\frac{1}{x}+1\right)^2} =$$

$$= \frac{\frac{1}{4x}\cdot 4\cdot\left(\frac{1}{x}+1\right) - \ln(4x)\cdot\left(-\frac{1}{x^2}\right)}{\left(\frac{1}{x}+1\right)^2}$$

- $\sin(\ln(2x))$

$$[\sin(\ln(2x))]' = \sin'(\ln(2x))\cdot(\ln(2x))' =$$

$$= \cos(\ln(2x))\cdot\frac{1}{2x}\cdot 2$$

- $\frac{\sqrt[8]{-x+2}}{ctg(2x)}$

$$\left[\frac{(-x+2)^{1/8}}{ctg(2x)}\right]' = \frac{\left[(-x+2)^{1/8}\right]'\cdot ctg(2x) - (-x+2)^{1/8}\cdot[ctg(2x)]'}{[ctg(2x)]^2} =$$

$$= \frac{\frac{1}{8}(-x+2)^{-7/8}\cdot(-1)\cdot ctg(2x) - (-x+2)^{1/8}\cdot\left[-\frac{1}{\sin^2(2x)}\cdot 2\right]}{[ctg(2x)]^2}$$

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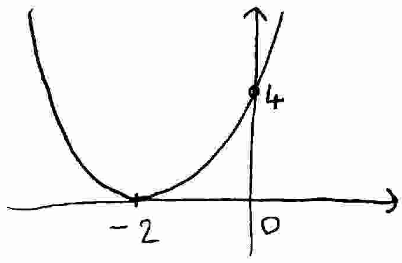
- Rajzold le az alabbi fuggvenyt! $f(x) = (x+2)^2$



- Szamold ki az alabbi fuggveny inverzet! $f(x) = 2\ln(x+5)$

$y = 2\ln(x+5)$ $\qquad$ $x = e^{y/2} - 5$

$\frac{y}{2} = \ln(x+5)$ $\qquad$ $f^{-1}(y) = e^{y/2} - 5$

$e^{y/2} = e^{\ln(x+5)} = (x+5)$ $\qquad$ $f^{-1}(x) = e^{x/2} - 5$

- Szamold ki a kovetkezo sorozat hatarerteket ahogy $n \to \infty$! $a_n = \frac{3n+1}{2n+4}$

$$\lim_{n\to\infty} \frac{3n+1}{2n+4} = \lim_{n\to\infty} \frac{3+1/n}{2+4/n} = \frac{3}{2}$$

- Szamold ki a kovetkezo fuggveny x szerinti derivaltjat! $f(x) = \frac{1}{34x+1}$

$$\left[(34x+1)^{-1}\right]' = -1 \cdot (34x+1)^{-2} \cdot 34$$

- Ird fel az alabbi fuggveny elsorendu Taylor sorat (linearis kozeliteset) az $x = 0$ pont korul! $f(x) = 1 + \sin(3x)$

$f(0) = 1 + \sin(3 \cdot 0) = 1$

$f'(x) = \cos(3x) \cdot 3$ $\qquad$ $f'(0) = \cos(3 \cdot 0) \cdot 3 = 3$

$$f(x) \approx T_1(x) = 1 + 3x$$