1 Sample Test 2.

1.1 A

1. Compute the $\int f(x)\,dx$ indefinite integrals of the following functions!

$$\sqrt[3]{2x^7} + \sqrt[3]{(2x)^7} + \frac{7}{x^7},\ x\sin(3x).$$

2. Solve the following differential equations!

$$y' = x, \quad y(2) = 3,$$
$$y' = e^{2x}, \quad y(2) = 3,$$

3. Solve the following differential equation!

$$y' = 4 - 2y, \quad y(2) = 3.$$

4. Compute the following definite integral!

$$\int_0^{\pi} \cos(2x)\,dx. \tag{1}$$

5. There are 10 black and 5 white balls in a box. Suppose that we DO put back the balls after the drawings. What is the chance of drawing firstly 3 white and then 2 black balls? What is the chance of drawing 3 white and then 2 black balls if the order is irrelevant?

6. There are 10 black and 5 white balls in a box. Suppose that we DO NOT put back the balls after the drawings. What is the chance of drawing firstly 3 white and then 2 black balls? What is the chance of drawing 3 white and then 2 black balls if the order is irrelevant?

7. Let us suppose that 50% of the voters vote for the Huge Party, 10% vote for the Small Party and the rest is undecided. Compute the expected value and the variance of the number of votes by a single voter in the cases of the Huge and Small Parties. Do the same in the case when a polling firm asks 1000 randomly chosen voters.

   What will be the typical error of measurement of the popularity of these parties?

8. Find the critical points of the following function and determine the critical point's type!

$$2x^2 + 3y^2 - 4x + 7.$$

9. Let

$$\phi_X(x) = \begin{cases} 1/3, & \text{if } x = 2 \\ 1/6, & \text{if} \\ 1/2, & \text{if} \end{cases}$$

   be the probability mass distribution of the random variable X. Plot the corresponding cumulative distribution function!

1.2 B

1. Compute the $\int f(x)\,dx$ indefinite integrals of the following functions!

$$\sqrt[3]{2x^7} + \sqrt[3]{(2x)^7} + \frac{7}{x^7},\ x\cos(3x).$$

2. Solve the following differential equations!

$$y' = 1, \quad y(2) = 3,$$
$$y' = x^2 - x, \quad y(2) = 3,$$

3. Solve the following differential equation!

$$y' = 6y + 12, \quad y(2) = 3.$$

4. Suppose that we toss a dice. Six numbers (from 1 to 6) can appear face up, but we do not yet know which one of them will appear. The sample space is: $\Omega = \{1, 2, 3, 4, 5, 6\}$. Define the events E and F as follows: $E = \{1, 3, 4\}$, $F = \{3, 4, 5, 6\}$. Prove that E and F are independent.

5. There are two urns containing colored balls. The first urn contains 50 red balls and 50 blue balls. The second urn contains 30 red balls and 70 blue balls. One of the two urns is randomly chosen (both urns have probability 50% of being chosen) and then a ball is drawn at random from one of the two urns. If a red ball is drawn, what is the probability that it comes from the first urn?

6. Compute the $f'_x, f'_y, f''_{xx}, f''_{xy}, f''_{yx}, f''_{yy}$ partial derivatives of the following function: $\sin(2x + y)\cos(3y)$.

7. Let

$$\phi_X(x) = \begin{cases} 1/3, & \text{if } 2 \leq x \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

be the probability density of the random variable X. Plot the corresponding cumulative distribution function!