

Test 2. Exercises

1. • Compute the $\int f(x) dx$ indefinite integrals of the following functions!

$$5, x, x^2 - 2, x^2 - x, \sqrt[3]{2x^7} + \sqrt[3]{(2x)^7} + \frac{7}{x^7}, \\ e^x + \sin(x), e^{3x} + \sin(3x).$$

- Find the general solutions of the following differential equations!

$$y'(x) = 5, y'(x) = x, y'(x) = x^2 - 2, y'(x) = x^2 - x, y'(x) = \sqrt[3]{2x^7} + \sqrt[3]{(2x)^7} + \frac{7}{x^7}, \\ y'(x) = e^x + \sin(x), y'(x) = e^{3x} + \sin(3x).$$

- Find the particular solutions of the following differential equations!

$$y'(x) = 5, y(1) = 4, \quad y'(x) = x, y(-1) = 7, \quad y'(x) = x^2 - 2, y(0) = 0.$$

2. Compute the $\int f(x) dx$ indefinite integrals of the following functions!

$$x \cdot e^x, \quad x \cdot \sin(x), \quad x \cdot \cos(x), \quad x^3 \ln(x), \quad 1 \cdot \ln(x), \\ x \cdot e^{-x}, \quad x \cdot \sin(2x), \quad x \cdot \cos(3x), \quad x^3 \ln(4x), \quad 1 \cdot \ln(5x).$$

3. (a) Compute $\int (1/5x)^3 + \exp(4x) - 1/(5x)^3 dx$!

(b) Compute $\int x \sin(4x) dx$ and $\int x \cos(-x) dx$!

(c) Compute $\int x^2 \ln(x) dx$ and $\int x^3 \ln(5x) dx$!

(d) Compute $\int (5x)^3 + \sin(4x) - \sqrt[3]{x^5} dx$!

4. Compute the area under the function $f(x)$ on the interval $[a, b]$!

(a) $f(x) = 7, [a, b] = [3, 9]; \quad f(x) = 2x, [a, b] = [3, 9]; \quad f(x) = e^{-2x}, [a, b] = [0, 4].$

(b) $f(x) = -7, [a, b] = [3, 9]; \quad f(x) = 2x, [a, b] = [9, 3]; \quad f(x) = -e^{-2x}, [a, b] = [0, 4].$

Discuss the signs of the corresponding definite integrals!

5. Solve the following differential equations!

(a) $y'(x) = 3; \quad y'(x) = x - 1; \quad y'(x) = e^{-3x}.$

(b) $y'(x) = 3, y(1) = 2; \quad y'(x) = x - 1, y(1) = 2; \quad y'(x) = e^{-3x}, y(1) = 2.$

(c) $y'(x) = 3y(x); \quad y'(x) = -3y(x), y(0) = 77; \quad y'(x) = -3y(x), y(1) = 77.$

6. Solve the $y'(x) = -3y(x) + 12$ differential equations!

(a) Find the equilibrium value y_f of the DE!

(b) What differential equation is satisfied by $\Delta y = y - y_f$?

(c) What is the general solution y_{gen} of the original DE?

7. Compute the $f'_x, f'_y, f''_{xx}, f''_{xy}, f''_{yx}, f''_{yy}$ partial derivatives of the following functions:

$$x^2 + y - 3, \quad x^3 y^{-5}, \quad x^3 (3y)^{-5}, \quad \sin(2x) \cos(3y).$$

8. The following functions have critical points at $(x, y) = (0, 0)$.

$$x^2 + y^2, \quad x^2 - y^2, \quad -x^2 + y^2, \quad xy.$$

Find the type of the critical points (preferably without calculation) !

9. Find the critical points of the following functions and determine their types!

$$2x^2 + 3y^2 - 4x + 7, \quad x^2 - y^2 - 2x + 2y, \quad x^2 + y^2 - 3xy, \quad xy - x - y - 1.$$