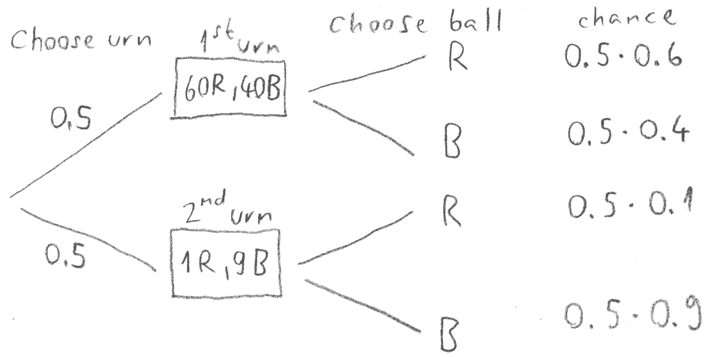
Math.Econ.Anal.MakeUp.Test.2. 15.dec.2.

1. (6+(2+2) points)
   A) There are two urns containing colored balls. The first urn contains 60 red balls and 40 blue balls. The second urn contains 1 red balls and 9 blue balls. One of the two urns is randomly chosen (both urns have probability 50% of being chosen), and then a ball is drawn at random from the chosen urn. If a red ball is drawn, what is the probability that it comes from the first urn?
   B) There are 10 black and 20 white balls in a box. Suppose that we DO NOT put back the balls after the drawings.
   Ba) What is the chance of drawing firstly 2 white and then 4 black balls?
   Bb) What is the chance of drawing 2 white and then 4 black balls if the order is irrelevant?

A 1



$$p(1^{st}\text{urn} \mid \text{Red drawn}) = \frac{0.5 \cdot 0.6}{0.5 \cdot 0.6 + 0.5 \cdot 0.1}$$

2 Bayes th: $$p(1^{st}\text{urn} \mid \text{Red}) = \frac{p(\text{Red} \mid 1^{st}\text{urn}) \cdot P(1^{st}\text{urn})}{p(\text{Red})} = \frac{0.6 \cdot 0.5}{0.5 \cdot 0.6 + 0.5 \cdot 0.1}$$

B a) $$p(WWBBBB) = \frac{20}{30} \cdot \frac{19}{29} \cdot \frac{10}{28} \cdot \frac{9}{27} \cdot \frac{8}{26} \cdot \frac{7}{25}$$

b) $$p(2W \text{ and } 4B) = p(WWBBBB) \cdot \binom{6}{2}$$

or $\binom{6}{4}$

or $$p(2W \text{ and } 4B) = \frac{\binom{20}{2}\binom{10}{4}}{\binom{30}{6}} = \frac{\binom{20}{2}\binom{10}{4}}{\binom{20+10}{2+4}}$$

2. (4+3+3 points)

A) Toss a fair coin two times, and let h be equal to 1, if two heads are tossed, otherwise h is zero. Compute $E[h]$ and $Var[h]$!

B) Solve the following differential equation! $y'(x) = 2y(x), \quad y(0) = 9;.$

C) Solve the following differential equation! $y'(x) = 2x, \quad y(0) = 9;.$

(A) $\Omega = \{HH, HT, TH, TT\}, \quad p(\{HH\}) = \ldots = p(\{TT\}) = \frac{1}{4}$

$h(HH) = 1, \quad h(HT) = h(TH) = h(TT) = 0$

$E(h) = \frac{1}{4} \cdot 1 + \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 0 = \frac{1}{4}$

$Var[h] = \frac{1}{4} \cdot \left(1 - \frac{1}{4}\right)^2 + \frac{1}{4} \cdot \left(0 - \frac{1}{4}\right)^2 + \frac{1}{4}\left(0 - \frac{1}{4}\right)^2 + \frac{1}{4}\left(0 - \frac{1}{4}\right)^2 = \frac{3}{8}$

(B) $y'(x) = 2y(x) \longrightarrow y_{gen}(x) = C \cdot e^{2x}$

$y(0) = 9 \longrightarrow C \cdot e^{2 \cdot 0} = 9 \longrightarrow C = 9 \longrightarrow y_{part}(x) = 9e^{2x}$

(C) $y'(x) = 2x \longrightarrow y_{gen}(x) = \int 2x\,dx = 2 \cdot \frac{x^2}{2} + C = x^2 + C$

$y(0) = 9 \longrightarrow 0^2 + C = 9 \longrightarrow C = 9 \longrightarrow y_{pard}(x) = x^2 + 9$

3. (4+6 points)

A) Compute the $f'_x, f'_y, f''_{yy}, f''_{yx}$ partial derivatives of the following function: $f(x,y) = x^3y^6 + y$.

B) Find the critical point of the following function and determine its type! $x^2 - 2xy - 2x + 8$.

(A) $f(x,y) = x^3y^6 + y$

$f'_x = 3x^2y^6$ $\qquad f''_{yx} = (f'_y)'_x = (x^3 \cdot 6y^5 + 1)'_x = 3x^2 \cdot 6y^5$

$f'_y = x^3 \cdot 6y^5 + 1$ $\qquad f''_{yy} = (f'_y)'_y = (x^3 \cdot 6y^5 + 1)'_y = x^3 \cdot 6 \cdot 5 \cdot y^4$

(B) $f(x,y) = x^2 - 2xy - 2x + 8$

$f'_x = 2x - 2y - 2 + 0 = 2x - 2y - 2$

$f'_y = 0 - 2x - 0 + 0 = -2x$

$f''_{xx} = (2x - 2y - 2)'_x = 2$

$f''_{yx} = f''_{xy} = (2x - 2y - 2)'_y = -2$

$f''_{yy} = (-2x)'_y = 0$

Location of crit point: $\left.\begin{array}{l} f'_x = 0 = 2x - 2y - 2 \\ f'_y = 0 = -2x \end{array}\right\} \to \begin{array}{l} y = -1 \\ x = 0 \end{array}$ $\qquad P_{crit} = (0, -1)$

Type of P_{crit}:

$\text{Hessian}(f) = \begin{pmatrix} 2 & -2 \\ -2 & 0 \end{pmatrix}$, $\quad (\text{Hessian}(f))(P_{crit}) = \begin{pmatrix} 2 & -2 \\ -2 & 0 \end{pmatrix}$

(1): $2 \cdot 0 - (-2) \cdot (-2) = -4 < 0$

Saddle point

4. (2+3+2+3 points)
Compute the following integrals:

(a) $\int \sqrt[9]{-x^5} + (5x)^6 + \frac{3}{2x}\,dx$

(b) $\int x \cdot \cos(5x)\,dx$

(c) $\int_2^5 1 - 2x\,dx$

Solve the following DE: $y'(x) = 3x^2, y(1) = 2$.

a) $\int \sqrt[9]{-x^5} + (5x)^6 + \frac{3}{2x}dx = \int (-x)^{5/9} + (5x)^6 + \frac{3}{2}\cdot\frac{1}{x}\,dx$

$$= \frac{\frac{(-x)^{14/9}}{14/9}}{-1} + \frac{\frac{(5x)^7}{7}}{5} + \frac{3}{2}\cdot \ln|x| + C$$

b) $\int \underbrace{x}_{g} \underbrace{\cos(5x)}_{f'}\,dx = \left| \begin{matrix} f' = \cos(5x) & g = x \\ f = \frac{\sin(5x)}{5} & g' = 1 \end{matrix} \right| = \frac{\sin(5x)}{5}\cdot x - \int \frac{\sin(5x)}{5}\cdot 1\,dx$

$$= \frac{\sin(5x)\cdot x}{5} - \frac{1}{5}\cdot\frac{-\cos(5x)}{5} + C$$

c) $\int_2^5 1-2x\,dx = \left[x - 2\cdot\frac{x^2}{2}\right]_2^5 = (5-5^2) - (2-2^2) = -18$

$y'(x) = 3x^2 \longrightarrow y_{gen}(x) = \int 3x^2\,dx = 3\cdot\frac{x^3}{3} + C = x^3 + C$

$y(1) = 2 \longrightarrow 1^3 + C = 2 \longrightarrow C = 1 \longrightarrow y_{part} = x^3 + 1$