# Math.Econ.Anal.Test.1. 15.oct.20.

1. (4+(1+1+2+1+1) points)

   (a) $\overline{m} = (4,4)^T$, $\overline{p} = (-4,4)^T$. Solve the equation $\alpha\overline{m} + \beta\overline{p} = (5,9)^T$ for $\alpha$ and $\beta$ !

   (b) $y = f(x) = 4 - 2x$.

      i. Where are the intersections of this line with the $x$ and $y$ axes?

      ii. What is the slope of this function?

      iii. Express $x$ with $y$ !

      iv. What is the $f^{-1}$ inverse of the function $f$ ?

      v. Plot $f$ and $f^{-1}$ together!

a) $\alpha\begin{pmatrix}4\\4\end{pmatrix} + \beta\begin{pmatrix}-4\\4\end{pmatrix} = \begin{pmatrix}5\\9\end{pmatrix} \longrightarrow \begin{pmatrix}4\alpha - 4\beta\\4\alpha + 4\beta\end{pmatrix} = \begin{pmatrix}5\\9\end{pmatrix}$

$\begin{matrix}4\alpha - 4\beta = 5\\4\alpha + 4\beta = 9\end{matrix} \longrightarrow 8\alpha = 14,\ \alpha = \frac{7}{4} \longrightarrow \beta = \frac{1}{2}$

$\beta$) i) $f(0) = 4$ $\qquad 4 - 2x = 0 \rightarrow x = 2$

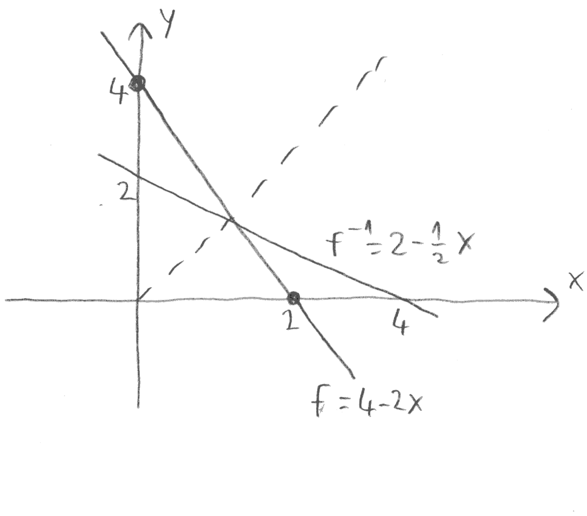
intersections: $(0,4)$ $\qquad (2,0)$

ii) slope: $-2$ $\qquad$ (coeff. of $x$, or $f'(x) = -2$)

iii) $4 - 2x = y \longrightarrow x = \frac{4-y}{2} = 2 - \frac{1}{2}y$

iv) $f^{-1}(x) = 2 - \frac{1}{2}x$

v)



2. ((2+3)+(3+1+1) points)

$1.2a_n - 5$

(a) Let $f(x) = 1.2 \cdot x - 5$. Find the fixed point of $f$!
If $a_0 = 99$ and $a_{n+1} = f(a_n) = \cancel{2a_n + 1}$, then how much is $a_n$ ?

(b) Let $a_n = \frac{2n+3}{4n+5}$. Is $a_n$ increasing or decreasing? (Prove it!)
Is $a_n$ convergent as $n \to \infty$?
If the answer is yes, what is the limit of $a_n$ ?

a) Fixed point: $1.2x_f - 5 = x_f \longrightarrow x_f = 25$

$$a_n = \underbrace{1.2^n \underbrace{(99-25)}_{\Delta a_0 = a_0 - x_f}}_{\Delta a_n = 1.2^n \Delta a_0 \text{ (linearized dynamics)}} + 25$$

b) $$a_{n+1} - a_n = \frac{2(n+1)+3}{4(n+1)+5} - \frac{2n+3}{4n+5} =$$

$$= \frac{(2n+5)(4n+5) - (4n+9)(2n+3)}{(4n+9)(4n+5)} = \frac{-2}{(4n+9)(4n+5)} < 0 \quad \text{for } n = 0, 1, 2, \ldots$$

so $a_n$ is monoton decreasing.

$$\lim_{n \to \infty} \frac{2n+3}{4n+5} = \lim_{n \to \infty} \frac{2 + 3/n}{4 + 5/n} = \frac{2}{4} = 0.5$$

3. (3+(2+2+3) points)

(a) Let $f(x) = 3x^2 + 4x$, $x_0 = 3$. What is the prediction of the linear approximation of $f$ around $x_0$ for the value of $f(x_0 + \Delta x)$ ?

(b) i. Compute $(x^{-2}\cos(4x))'$ !

ii. Compute $(\sin(4x^3 + 3))'$ !

iii. Compute $\left(\sqrt[5]{3x^2} + \frac{4}{(3x)^6} + \ln(3x)\right)'$ !

a) $f(x) = 3x^2 + 4x \qquad f(3) = 39$

$f'(x) = 6x + 4 \qquad f'(3) = 22$

So $f(3+\Delta x) \approx f(3) + f'(3)\Delta x = 39 + 22\Delta x$

b) $\left[x^{-2}\cos(4x)\right]' = (x^{-2})'\cos(4x) + x^{-2}\cdot[\cos(4x)]' =$

$= -2x^{-3}\cdot\cos(4x) + x^{-2}\cdot(-\sin(4x)\cdot 4)$

$\left[\sin(4x^3+3)\right]' = \sin'(4x^3+3)\cdot(4x^3+3)' =$

$= \cos(4x^3+3)\cdot 12x^2$

$\left[\sqrt[5]{3x^2} + \frac{4}{(3x)^6} + \ln(3x)\right]' = \left[\sqrt[5]{3}\cdot x^{2/5} + 4\cdot(3x)^{-6} + \ln(3x)\right]' =$

$= \sqrt[5]{3}\cdot\frac{2}{5}x^{-3/5} + 4\cdot(-6)(3x)^{-7}\cdot 3 + \underbrace{\frac{1}{3x}\cdot 3}_{= \frac{1}{x}}$

$= \frac{1}{x}$, as $\ln(3x) = \ln x + \ln 3$

4. (7+3 points)

(a) Study the monotonicity, convexity and the local extremal values of the function $f(x) = x - x^3$ ! Draw the graphs of $f$ and $f'$ in the same coordinate system!

$f(x) = x - x^3$

1p $\quad f'(x) = 1 - 3x^2$

1p $\quad f''(x) = -6x$

1 Extremal val.: $f'(x_{extr}) = 0 \longrightarrow 1 - 3x^2 = 0$

1p $\quad x_1 = -\frac{1}{\sqrt{3}} \qquad x_2 = +\frac{1}{\sqrt{3}}$

type: $\quad f''(x_{extr}) \begin{matrix} <0 & \text{MAX} \\ >0 & \text{MIN} \end{matrix}$

1p $\quad f''\left(-\frac{1}{\sqrt{3}}\right) = -6\cdot\left(-\frac{1}{\sqrt{3}}\right) > 0 \qquad f''\left(\frac{1}{\sqrt{3}}\right) = -6\cdot\frac{1}{\sqrt{3}} < 0$

local MIN. $\qquad$ MAX.

1p $\quad f$ increasing on $\left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$, decreasing on $\left(-\infty, -\frac{1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}, \infty\right)$

2 Convexity: $\quad f''(x) > 0$ convex $\cup$

$f''(x) < 0$ concave $\cap$

2p $\quad f''(x) = -6x \begin{matrix} >0 & \text{if } x<0 \\ <0 & \text{if } x>0 \end{matrix}$ So $f$ convex on $(-\infty, 0]$, concave on $[0, \infty)$

3p