Math.Econ.Anal.Quiz.4. 15.nov.23.

1. Suppose that we toss a dice. Six numbers (from 1 to 6) can appear face up with equal probabilities. The sample space is: $\Omega = \{1,2,3,4,5,6\}$. Define the events E and F as follows: $E = \{1,2,5,6\}$. $F = \{1,3,5\}$. Are E and F independent? Justify your answer! (2)

2. There are two urns containing colored balls. The first urn contains 10 red balls and 90 blue balls. The second urn contains 80 red balls and 20 blue balls. One of the two urns is randomly chosen (both urns have probability 50% of being chosen), and then a ball is drawn at random from the chosen urns. If a red ball is drawn, what is the probability that it comes from the first urn? (4)

3. There are 9 black and 4 white balls in a box. Suppose that we DO NOT put back the balls after the drawings. What is the chance of drawing firstly 2 white and then 3 black balls? What is the chance of drawing 2 white and then 3 black balls if the order is irrelevant? (3)

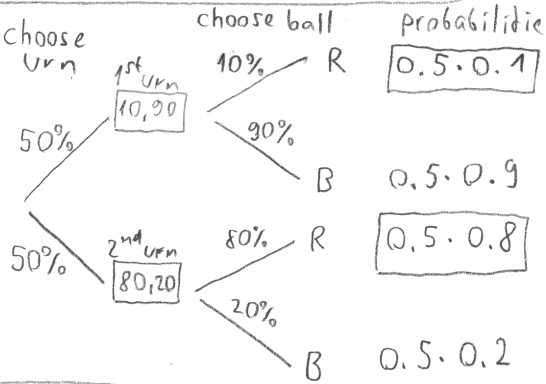
4. There are 9 black and 4 white balls in a box. Suppose that we DO put back the balls after the drawings. What is the chance of drawing firstly 2 white and then 3 black balls? What is the chance of drawing 2 white and then 3 black balls if the order is irrelevant? (3)

(1) $E = \{1,2,5,6\}$, $F = \{1,3,5\}$, $E \cap F = \{1,5\}$

$p(E) = \frac{4}{6}$ $\quad p(F) = \frac{3}{6}$ $\quad p(E \cap F) = \frac{2}{6}$

$p(E) \cdot p(F) = \frac{4}{6} \cdot \frac{3}{6} = \frac{2}{6} = p(E \cap F)$ so E and F are independent.

(2)



$p(1^{st}\text{ urn} \mid \text{Red}) = \frac{0.5 \cdot 0.1}{0.5 \cdot 0.1 + 0.5 \cdot 0.8} = \frac{1}{9}$

or apply Bayes th.

$p(1^{st}\text{ urn} \mid \text{Red drawn}) = \frac{p(\text{Red drawn} \mid 1^{st}\text{ urn}) \cdot p(1^{st}\text{ urn})}{p(\text{Red drawn})}$

$= \frac{10\% \cdot 50\%}{50\% \cdot 10\% + 50\% \cdot 80\%}$

(3) a) $p(WWBBB) = \frac{4}{13} \cdot \frac{3}{12} \cdot \frac{9}{11} \cdot \frac{8}{10} \cdot \frac{7}{9}$

b) $p(\text{2 white and 3 black}) = p(WWBBB) \cdot \binom{5}{2} = \frac{4}{13} \cdot \frac{3}{12} \cdot \frac{9}{11} \cdot \frac{8}{10} \cdot \frac{7}{9} \cdot \frac{5 \cdot 4}{1 \cdot 2}$ $\left(= \frac{\binom{4}{2}\binom{9}{3}}{\binom{13}{5}} \right)$ hypergeometric dist.

(4) a) $p(WWBBB) = \left(\frac{4}{13}\right)^2 \cdot \left(\frac{9}{13}\right)^3$

b) $p(\text{2 white and 3 black}) = p(WWBBB) \cdot \binom{5}{2} = \binom{5}{2} \cdot \left(\frac{4}{13}\right)^2 \cdot \left(\frac{9}{13}\right)^3$

↑ order is irrelevant