Math.Econ.Anal.Quiz.3. 15.nov.9.

1. Compute $\int 2x\sin(-3x)\,dx$. (3p)
2. Solve the following differential equation! $y'(x) = 2y(x),\quad y(0) = 9$;. (3p)
3. Compute the f'_x, f'_y partial derivatives of the following function: $x^3 + x^2y + y$. (2p)
4. Find the critical point of the following function and determine its type! $x^2 - xy - 2x$. (4p)

① $\int \underbrace{2x}_{g}\,\underbrace{\sin(-3x)}_{f'}\,dx = \left|\begin{array}{ll} f' = \sin(-3x) & g = 2x \\ f = \frac{-\cos(-3x)}{-3} & g' = 2 \end{array}\right| = \underbrace{\frac{-\cos(-3x)}{-3}}_{f}\cdot\underbrace{2x}_{g} - \int \underbrace{\frac{-\cos(-3x)}{-3}}_{f}\cdot\underbrace{2}_{g'}\,dx$

$= \frac{-\cos(-3x)}{-3}\cdot 2x - \frac{-\sin(-3x)}{(-3)^2}\cdot 2 + C$

$\boxed{\int f'g = fg - \int fg'}$

as $(e^{2x})' = 2\cdot e^{2x}$

by linearity of the DE

② $y' = 2y \xrightarrow{\text{general sol.}} y_{gen}(x) = C\cdot e^{2x}$

$y(0) = 9 \longrightarrow C\cdot e^{2\cdot 0} = 9 \longrightarrow C = 9 \xrightarrow{\text{part. sol.}} y_{part}(x) = 9e^{2x}$

③ $(x^3 + x^2y + y)'_x = 3x^2 + 2x\cdot y + 0$

$(x^3 + x^2y + y)'_y = 0 + x^2 + 1$

④ $f(x,y) = x^2 - xy - 2x$

$f'_x = 2x - y - 2$

$f'_y = 0 - x - 0$

Location of the critical point P_{crit}:

$\left.\begin{array}{l} f'_x = 0 = 2x - y - 2 \\ f'_y = 0 = -x \quad \rightarrow x = 0 \end{array}\right\} \rightarrow y = -2 \rightarrow P_{crit} = (0,-2)$

Type of P_{crit}:

$f''_{xx} = (2x - y - 2)'_x = 2$

$f''_{yx} = f''_{xy} = (2x - y - 2)'_y = -1$

$f''_{yy} = (-x)'_y = 0$

$\begin{pmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix} = \text{Hessian}(f)$

$[\text{Hessian}(f)](P_{crit}) = \begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix}$

$2\cdot 0 - (-1)\cdot(-1) = -1 < 0 \rightarrow$ saddle point