Math.Econ.Anal.Quiz.3.exercises 15.nov.9.

1. (a) Compute $\int (1/5x)^3 + \exp(4x) - 1/(5x)^3\, dx$!
   (b) Compute $\int x \sin(4x)\, dx$ and $\int x \cos(-x)\, dx$!
   (c) Compute $\int x^2 \ln(x)\, dx$ and $\int x^3 \ln(5x)\, dx$!
   (d) Compute $\int (5x)^3 + \sin(4x) - \sqrt[3]{x^5}\, dx$!

2. Compute the area under the function $f(x)$ on the interval $[a, b]$!
   (a) $f(x) = 7,\ [a, b] = [3, 9]; \quad f(x) = 2x,\ [a, b] = [3, 9]; \quad f(x) = e^{-2x},\ [a, b] = [0, 4].$
   (b) $f(x) = -7,\ [a, b] = [3, 9]; \quad f(x) = 2x,\ [a, b] = [9, 3]; \quad f(x) = -e^{-2x},\ [a, b] = [0, 4].$

   Discuss the signs of the corresponding definite integrals!

3. Solve the following differential equations!
   (a) $y'(x) = 3; \quad y'(x) = x - 1; \quad y'(x) = e^{-3x}.$
   (b) $y'(x) = 3,\ y(1) = 2; \qquad y'(x) = x - 1,\ y(1) = 2; \qquad y'(x) = e^{-3x},\ y(1) = 2.$
   (c) $y'(x) = 3y(x); \quad y'(x) = -3y(x),\ y(0) = 77; \quad y'(x) = -3y(x),\ y(1) = 77.$

4. Solve the $y'(x) = -3y(x) + 12$ differential equations!
   (a) Find the equilibrium value y_f of the DE!
   (b) What differential equation is satisfied by $\Delta y = y - y_f$?
   (c) What is the general solution y_{gen} of the original DE?

5. Compute the $f'_x, f'_y, f''_{xx}, f''_{xy}, f''_{yx}, f''_{yy}$ partial derivatives of the following functions:

$$x^2 + y - 3, \quad x^3 y^{-5}, \quad x^3 (3y)^{-5}, \quad \sin(2x)\cos(3y).$$

6. The following functions have critical points at $(x, y) = (0, 0)$.

$$x^2 + y^2, \quad x^2 - y^2, \quad , -x^2 + y^2, \quad xy.$$

   Find the type of the critical points!

7. Find the critical points of the following functions and determine their types!

$$2x^2 + 3y^2 - 4x + 7, \quad x^2 - y^2 - 2x + 2y, \quad , x^2 + y^2 - 3xy, \quad xy - x - y - 1.$$