Math.Econ.Anal.Quiz.2. 15.oct.5.

1. Compute $\lim_{n\to\infty}\left(1-\frac{6}{5n}\right)^{3n-2}$!
2. Let $f(x)=3x^2-5x,\ x_0=7$. Compute $\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$!
3. Let $f(x)=3x^2-5x,\ x_0=7$. What is the prediction of the linear approximation of f around x_0 for the value of $f(7+\Delta x)$?
4. Compute $((x+3)\cos(4x))'$!

① $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)=e^x$

$$\lim_{n\to\infty}\left(1-\frac{6}{5n}\right)^{3n-2}=\lim_{n\to\infty}\left[\left(1+\frac{-6/5}{n}\right)^n\right]^3\cdot\left(1-\frac{6}{5n}\right)^{-2}$$

$$=\left[e^{-6/5}\right]^3\cdot 1^{-2}=e^{-18/5}$$

② $f(x_0+\Delta x)=3(7+\Delta x)^2-5(7+\Delta x)=3\cdot(7^2+2\cdot 7\cdot\Delta x+\Delta x^2)-5(7+\Delta x)$

$$\frac{f(7+\Delta x)-f(7)}{\Delta x}=\frac{[3\cdot(7^2+2\cdot 7\cdot\Delta x+\Delta x^2)-5(7+\Delta x)]-[3\cdot 7^2-5\cdot 7]}{\Delta x}$$

$$=\frac{3\cdot 2\cdot 7\,\Delta x+3\Delta x^2-5\Delta x}{\Delta x}=(3\cdot 2\cdot 7-5)+3\Delta x=37+3\Delta x$$

③ $f(x)=3x^2-5x \qquad f(7)=3\cdot 7^2-5\cdot 7=112$

$f'(x)=6x-5 \qquad f'(7)=6\cdot 7-5=37$

$f(7+\Delta x)\approx 112+37\Delta x$

④ $(fg)'=f'g+fg'$

$$[(x+3)\cos(4x)]'=(x+3)'\cos(4x)+(x+3)[\cos(4x)]'=$$

$$=1\cdot\cos(4x)+(x+3)[-\sin(4x)\cdot 4]$$