

Math. Econ. Anal. Quiz. 1. 15. sept. 21.

1. $\underline{m} = (0, 2)^T$, $\underline{p} = (3, 1)^T$. Solve $\alpha \underline{m} + \beta \underline{p} = (10, 4)^T$ for α and β !

2. How much is $|(3, 4, 5)^T|$? How much is $(3, 1, 4, 5)^T(1, 3, 4, 5)^T$?

3. Let $\underline{r}_0 = (1, 1, 1)^T$ and $\underline{n} = (2, 2, 2)^T$. Find an equation of the plane that contains \underline{r}_0 and has normal vector \underline{n} !

Express the third coordinate $z(x, y, z)$ of the point (x, y, z) of the plane with x and y !

4. Let $f(x) = 3x + 4$. If $a_0 = 13$ and $a_{n+1} = f(a_n) = 3a_n + 4$, then how much is a_n ?

Solution:

① $\alpha \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix} \Rightarrow \begin{pmatrix} 3\beta \\ 2\alpha + \beta \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$

$\Rightarrow \begin{cases} 3\beta = 10 \\ 2\alpha + \beta = 4 \end{cases} \Rightarrow \begin{cases} \beta = 10/3 \\ \alpha = 1/3 \end{cases}$

3 points

② $\left| \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \right| = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50}$

$\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = 3 \cdot 1 + 1 \cdot 3 + 4 \cdot 4 + 5 \cdot 5 = 47$

2 points

③ $\underline{n}(\underline{r} - \underline{r}_0) = 0 \Rightarrow \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \cdot \left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right] = 0$

$\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} x-1 \\ y-1 \\ z-1 \end{pmatrix} = 0 \Rightarrow 2(x-1) + 2(y-1) + 2(z-1) = 0$

3 points

$\Rightarrow 2x + 2y + 2z - 6 = 0 \Rightarrow z(x, y) = 3 - x - y$

④ Fixed point x_f : $f(x_f) = x_f \Rightarrow 3x_f + 4 = x_f \Rightarrow -2 = x_f$

$a_n = 3^n(13 - (-2)) + (-2) = 3^n(17) - 2$

3 points