Correctional Test 1. Econ.Anal 14.Dec.   NEPTUN: Name:

1. A. Compute $C = \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. $= 3 \cdot 1 + 1 \cdot 3 = 6$ $\boxed{1}$

B. Compute the square of the Eucledian length of $\{1, 1, 1, 0\}^T$! $\left\| \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\| = \sqrt{1^2+1^2+1^2+0^2} = \sqrt{3}$ $\boxed{2}$

C. Compute $\begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}^{-1}$. $\boxed{2}$ $\begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x & u \\ y & v \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\boxed{1}$ $\begin{pmatrix} 4x+0y & 4u+0v \\ 1x+2y & 1u+2v \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$4x+0y=1$ $\boxed{2}$ $4u+0v=0$ Inverse: $\boxed{2}$

$1x+2y=0$ $\quad 1u+2v=1$ $\quad \begin{pmatrix} 1/4 & 0 \\ -1/8 & 1/2 \end{pmatrix}$

$x = \frac{1}{4}, y = -\frac{1}{8}$ $\quad u = 0, v = \frac{1}{2}$

2. A. Let $f(x) = 1 - 3x$. Compute $\frac{f(5+\Delta x_n) - f(5)}{\Delta x_n}$! What is the limit of this fraction if $\Delta x_n = 1/n$? What is $f'(5)$?

$\boxed{1}$ lim B. Study the monotonicity and the limit of the following sequences!

$-\frac{1}{5}$ a) $\frac{4-n}{5n+6}$, $\boxed{1}$ 0 b) $\frac{3n}{5}$. $\quad \frac{4-(n+1)}{5(n+1)+6} - \frac{4-n}{5n+6} = \frac{3-n}{5n+11} - \frac{4-n}{5n+6} = \frac{-23}{(5n+11)(5n+6)} < 0$ decreasing $\boxed{2}$ if $n = 1, 2, 3 \dots$ $\boxed{1}$

$\frac{3n}{5}$ monoton decreasing $\boxed{1}$

3.A. Compute the limit of the following sequence! $a_n = \left(1 + \frac{1}{3n}\right)^{3n-7} \left(\frac{2n^2}{3n^2+1}\right)$.

B. Let $\phi(x) = -3x + 2$, $x_0 = 2$, $x_{n+1} = \phi(x_n)$. What are ϕ^{-1} and $\phi^n(x_0) = x_n$?

1. Compute ϕ^{-1} ! $\quad y = -3x+2, \; x = \frac{y-2}{-3}, \; f^{-1}(x) = \frac{x-2}{-3} = \frac{2}{3} - \frac{x}{3}$ $\boxed{2}$

2. Find the fixed point x_f of ϕ ! $\quad x_f = -3x_f + 2 \longrightarrow x_f = 1/2$ $\boxed{1}$

3. Compute x_n ! $\quad x_n = (-3)^n \cdot \left(2 - \frac{1}{2}\right) + \frac{1}{2}$ $\boxed{2}$

$a_n = \left[\left(1 + \frac{1/3}{n}\right)^n\right]^3 \cdot \left(1 + \frac{1/3}{n}\right)^{-7} \cdot \frac{2}{3 + 1/n^2} \longrightarrow \left[e^{1/3}\right]^3 \cdot 1^{-7} \cdot \frac{2}{3} = \frac{2e}{3}$ $\boxed{3}$ $\boxed{2}$

4. A. Let $\bar{v}_1 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\bar{v}_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 8 \end{pmatrix} = \alpha \bar{v}_1 + \beta \bar{v}_2$. Compute $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$! $\begin{pmatrix} 2 \\ 8 \end{pmatrix} = \begin{pmatrix} 2\alpha + 2\beta \\ -2\alpha + 2\beta \end{pmatrix}$ $\quad \alpha = -1.5$, $\beta = 2.5$ $\boxed{2}$

B. Let T be a 2×2 matrix formed by the transition probabilities of a two state (labeled by 1 and 2) stochastic system, where
$T(1 \leftarrow 1) = T_{11} = 0$, $T(2 \leftarrow 1) = T_{21} = 1$, $T(1 \leftarrow 2) = T_{12} = 0.2$, $T(2 \leftarrow 2) = T_{22} = 0.8$.

1. Find an eigenvector $\bar{v}_1$ corresponding to the eigenvalue $\lambda_1 = 1$! (This is the equilibrium state.)

2. Find the eigenvalue λ_2 of T corresponding to the eigenvector $\bar{v}_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$!

3. Calculate α and β in $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha \bar{v}_1 + \beta \bar{v}_2$!

4. Calculate $T(\alpha \bar{v}_1 + \beta \bar{v}_2)$, $T^2(\alpha \bar{v}_1 + \beta \bar{v}_2)$, etc.

2 A: $\frac{(1 - 3(5 + \Delta x_n)) - (1 - 3 \cdot 5)}{\Delta x_n} \overset{\boxed{2}}{=} -3$

$\lim_{n \to \infty} -3 = -3$, so $f'(5) = -3$ $\boxed{1}$ $\boxed{1}$

① $\begin{pmatrix} 0 & 0.2 \\ 1 & 0.8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1 \cdot \begin{pmatrix} x \\ y \end{pmatrix}$ $\quad 0x + 0.2y = x$, $\; 1x + 0.8y = y$ $\boxed{3}$

eigenvects: $\begin{pmatrix} x \\ 0.2x \end{pmatrix}$, $\bar{V}_1 = \begin{pmatrix} 5/6 \\ 1/6 \end{pmatrix}$

② $\begin{pmatrix} 0 & 0.2 \\ 1 & 0.8 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0.4 \\ -0.4 \end{pmatrix} = -0.2 \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

so $\lambda_2 = -0.2$ $\boxed{2}$

③ $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha \cdot 5/6 + \beta \cdot 2 \\ \alpha \cdot 1/6 + \beta \cdot (-2) \end{pmatrix} \longrightarrow \alpha = 1, \; \beta = -5/12$ $\boxed{1}$

④ $T^n \left[1 \cdot \begin{pmatrix} 5/6 \\ 1/6 \end{pmatrix} + \left(-\frac{5}{12}\right) \begin{pmatrix} -2 \\ 2 \end{pmatrix}\right] = 1 \cdot 1^n \cdot \bar{V}_1 + \left(-\frac{5}{12}\right) \cdot (-0.2)^n \bar{V}_2$ $\boxed{2}$

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Correctional.Test.2. 14.Dec.11. Name: **Neptun:**

I. $\boxed{5\times 2}$

1. Compute the derivatives of the following functions!

- $\sqrt[5]{x^6} - \frac{1}{\sqrt[6]{(3x)^5}} + \sin(3x-1)$ $= x^{6/5} - (3x)^{-5/6} + \sin(3x-1) \rightarrow \frac{6}{5}x^{1/5} - \left(-\frac{5}{6}\right)(3x)^{-11/6}\cdot 3 + \cos(3x-1)\cdot 3$
- $\sin x \ln(-x-1)$ $\longrightarrow \cos x \cdot \ln(-x-1) + \sin x \cdot \frac{1}{-x-1}\cdot(-1)$
- $\frac{\sin(2x)}{x^{-2}-1}$ $\longrightarrow \frac{\cos(2x)\cdot 2\cdot(x^{-2}-1) - \sin(2x)\cdot[-2\cdot x^{-3}]}{[x^{-2}-1]^2}$ $\boxed{2}$

2. Compute the $\int f(x)\,dx$ indefinite integrals of the following $f(x)$ functions!

- $\cos(-x) + \sin(3x)$ $\longrightarrow \frac{\sin(-x)}{-1} + \frac{-\cos(3x)}{3} + C$
- $(3x)^{-2} - 2x - 1$ $\longrightarrow \frac{(3x)^{-1}}{-1}\cdot\frac{1}{3} - 2\cdot\frac{x^2}{2} - x + C$

II.

1. Compute the $\int x\sin(-3x)\,dx$!

$\left| \begin{matrix} f' = \sin(-3x) & g = x \\ f = \frac{-\cos(-3x)}{-3} & g' = 1 \end{matrix} \right| = \frac{-\cos(-3x)}{-3}x - \underbrace{\int \frac{-\cos(-3x)}{-3}\cdot 1\,dx}$ $\boxed{1}$

$- \frac{\sin(3x)}{(-3)^2} + C$ $\boxed{1}$

2. Find the local extremal values of the following function: $f(x) = -x^3 - x^2$!

$f' = -3x^2 - 2x$
$f'' = -6x - 2$ $\boxed{2}$

$f' = 0$
$-x(3x+2) = 0$

$\boxed{2}$ $x_1 = 0$ $f''(0) = -6\cdot 0 - 2 = -2 < 0$ $\boxed{2}$ MAX
$x_2 = -2/3$ $f''\left(-\frac{2}{3}\right) = -6\cdot\frac{-2}{3} - 2 = 2 > 0$ MIN

III.

Solve the following differential equations!

- $y' = 13, \quad y(1) = 13,$

$\boxed{2}$ $y = \int 13\,dx = 13x + C$
$13\cdot 1 + C = 13 \rightarrow C = 0,\ y = 13x$

- $y' = \sin(2x), \quad y(2) = 13,$

$y = \int \sin(2x)\,dx = \frac{-\cos(2x)}{2} + C$
$\boxed{3}$ $-\frac{\cos(2\cdot 2)}{2} + C = 13$
$C = 13 + \frac{\cos 4}{2},\ y = \frac{-\cos(2x)}{2} + 13 + \frac{\cos 4}{2}$

- $y' = 1 - 2y, \quad y(2) = 3,$

fixed point: $0 = y' = 1 - 2y \rightarrow y_f = \frac{1}{2}$ $\Delta y = y - \frac{1}{2}$, $y = \Delta y + \frac{1}{2}$, $\Delta y' = y'$
$\boxed{5}$ $\Delta y' = -2\Delta y \rightarrow \Delta y = C\cdot e^{-2x} \rightarrow y = C\cdot e^{-2x} + \frac{1}{2}$ $C\cdot e^{-2\cdot 2} + \frac{1}{2} = 3 \rightarrow C = \frac{3 - 1/2}{e^{-4}}$

IV.

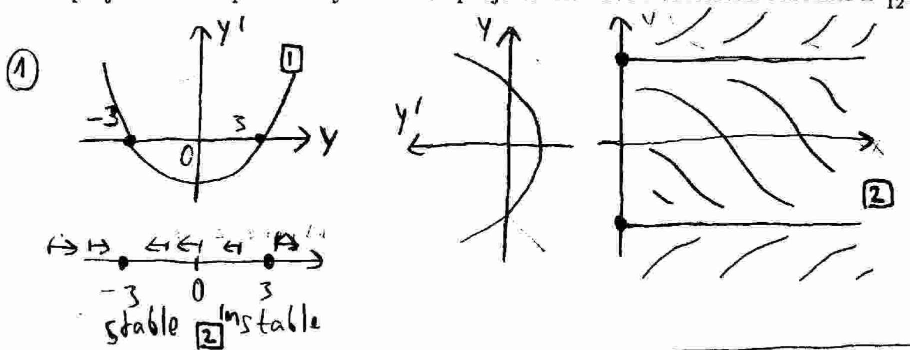
(1) • Plot the solution set of the following differential equation! Find the equilibrium positions and determine their stability!

$$y' = y^2 - 9.$$

(2) • There are 8 black and 5 white balls in a box. Suppose that we DO NOT put back the balls after the drawings. What is the chance of drawing firstly 2 white and then 3 black balls? What is the chance of drawing 2 white and then 3 black balls if the order is irrelevant?

(3) • A firm undertakes two projects, A and B. The probabilities of having successful outcomes are $\frac{1}{4}$ for project A and $\frac{1}{3}$ for project B. The probability that both projects will have a successful outcome is $\frac{1}{12}$. Are the two outcomes independent?

(1)



(3) $p(A) = \frac{1}{4}$, $p(B) = \frac{1}{3}$
if independent:
$p(A\cap B) = p(A)\,p(B)$
true, as $\frac{1}{4}\cdot\frac{1}{3} = \frac{1}{12}$ $\boxed{2}$

(2) First 2W, then 3B: $p(WWBBB) = \frac{5}{13}\cdot\frac{4}{12}\cdot\frac{8}{11}\cdot\frac{7}{10}\cdot\frac{6}{9}$ $\boxed{1}$

$p(2W \text{ and } 3B)$: $p(WWBBB)\cdot\binom{5}{2} =$
$\frac{\binom{5}{2}\cdot\binom{8}{3}}{\binom{13}{5}}$ $\boxed{2}$

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