Test 1. Econ.Anal 14.Oct. NEPTUN: Name:

1. A. Compute $C = \begin{pmatrix} -3 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

B. Compute the square of the Eucledian length of $\{1, 1, -1, 0\}^T$!

C. Compute $\begin{pmatrix} 4 & 3 \\ 0 & 2 \end{pmatrix}^{-1}$.

2. A. Let $f(x) = 3x^2 + 1$. Compute $\frac{f(5+\Delta x_n) - f(5)}{\Delta x_n}$! What is the limit of this fraction if $\Delta x_n = 1/n$? What is $f'(5)$?

B. Study the monotonicity and the limit of the following sequences!

a) $\frac{4-1}{5n+6}$, b) $\frac{3n}{5}(-1)^n$.

3.A. Compute the limit of the following sequence! $a_n = \left(1 - \frac{1}{3n}\right)^{-3n+7} \left(\frac{2n^2}{3n^2+1}\right)$.

B. Let $\phi(x) = 3x - 2$, $x_0 = 2$, $x_{n+1} = \phi(x_n)$. What are ϕ^{-1} and $\phi^n(1) = x_n$?

1. Compute ϕ^{-1} !
2. Find the fixed point x_f of ϕ !
3. Compute x_n !

4. A. Let $\bar{v}_1 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\bar{v}_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 12 \\ 82 \end{pmatrix} = \alpha \bar{v}_1 + \beta \bar{v}_2$. Compute $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$!

B. Let T be a 2×2 matrix formed by the transition probabilities of a two state (labeled by 1 and 2) stochastic system, where
$T(1 \leftarrow 1) = T_{11} = 1$, $T(2 \leftarrow 1) = T_{21} = 0$, $T(1 \leftarrow 2) = T_{12} = 0.2$, $T(2 \leftarrow 2) = T_{22} = 0.8$.

1. Find an eigenvector $\bar{v}_1$ corresponding to the eigenvalue $\lambda_1 = 1$! (This is the equilibrium state.)
2. Find the eigenvalue λ_2 of T corresponding to the eigenvector $\bar{v}_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$!
3. Calculate α and β in $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha \bar{v}_1 + \beta \bar{v}_2$!
4. Calculate $T(\alpha \bar{v}_1 + \beta \bar{v}_2)$, $T^2(\alpha \bar{v}_1 + \beta \bar{v}_2)$, etc.