

I.

A. Compute the derivatives of the following functions!

- $\sqrt[3]{2x^4} - \frac{2}{x^2} + \sin(3x)$
- $\cos x \cdot \ln(2x - 1)$
- $\frac{\sin(2x)}{1+3x^2}$

B. Compute the following definite integrals!

- $\int_0^1 e^{-x} + \sin(-3x) dx$
- $\int_1^2 \sqrt{(2x)^3} - 5/x^2 dx$

2.

- Plot the solution set of the following differential equation! Find the equilibrium position and determine its stability!

$$y' = y - 1.$$

- There are 7 black and 5 white balls in a box. Suppose that we DO put back the balls after the drawings. What is the chance of drawing firstly 3 white and then 3 black balls? What is the chance of drawing 3 white and then 3 black balls if the order is irrelevant?
- Suppose that we roll a dice. Six numbers (from 1 to 6) can appear face up, but we do not yet know which one of them will appear. The sample space is:  $\Omega = \{1, 2, 3, 4, 5, 6\}$ . Define the events  $E$  and  $F$  as follows:  $E = \{1, 2\}$ ,  $F = \{3, 4, 5\}$ . Are  $E$  and  $F$  independent? Prove your answer!

3.

A. Let  $f(x) = x + 7$ . Compute  $\frac{f(2+\Delta x_n) - f(2)}{\Delta x_n}$  ! What is the limit of this fraction if  $\Delta x_n = 1/n$  ? How much is  $f'(2)$  ?

B. Study the monotonicities and the limits of the following sequences!

a)  $\frac{n}{2n-1}$ ,      c)  $\frac{n}{2n-1}(-1)^n$ .

4.

A. Compute  $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$  and  $\begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$ B. Compute the Euclidian length of  $\{2, 3, 0, -2\}^T$  !

C. Solve the following differential equations!

$$y' = 2x, \quad y(2) = 5, \tag{1}$$

$$y' = -y, \quad y(2) = 3, \tag{2}$$

$$y' = -y + 5. \tag{3}$$