Exam. Econ.Anal. 14.Dec.19. NEPTUN: Name:

I.

A. Compute the derivatives of the following functions!

- $\sqrt[3]{x^7} - \frac{3}{\sqrt[6]{x^6}} + \cos(3x)$
- $\ln x \sin(-x-1)$
- $\frac{\sin(2x)}{1-x^2}$

B. Compute the following definite integrals!

- $\int_0^1 e^{-x} + \sin(3x)\, dx$
- $\int_0^1 x^2 - 2x\, dx$

2.

- Plot the solution set of the following differential equation! Find the equilibrium positions and determine their stability!
$$y' = -y^2 + y.$$
- There are 9 black and 4 white balls in a box. Suppose that we DO NOT put back the balls after the drawings. What is the chance of drawing firstly 2 white and then 3 black balls? What is the chance of drawing 2 white and then 3 black balls if the order is irrelevant?
- A firm undertakes two projects, A and B. The probabilities of having a successful outcome are $\frac{1}{4}$ for project A and $\frac{1}{3}$ for project B. The probability that both projects will have a successful outcome is $\frac{1}{16}$. Are the two outcomes independent?

3.

A. Let $f(x) = 3x^2 + 7$. Compute $\frac{f(2+\Delta x_n)-f(2)}{\Delta x_n}$! What is the limit of this fraction if $\Delta x_n = 1/n$? How much is $f'(2)$?

B. Study the monotonicities and the limits of the following sequences!

a) $\frac{2n}{3n+6}$, b) $\frac{3}{n^2+1}(-1)^n$.

4.

A. Compute $C = \begin{pmatrix} -1 \\ -2 \end{pmatrix} \begin{pmatrix} 1 & -3 \end{pmatrix}$.

B. Compute the Eucledian length of $\{2, 0, -1, -2\}^T$!

C. Let T be a 2×2 matrix formed by the transition probabilities of a two state (labeled by 1 and 2) stochastic system, where
$T(1 \leftarrow 1) = T_{11} = 0$, $T(2 \leftarrow 1) = T_{21} = 1$, $T(1 \leftarrow 2) = T_{12} = 0.5$, $T(2 \leftarrow 2) = T_{22} = 0.5$.

1. Find an eigenvector $\bar{v}_1$ corresponding to the eigenvalue $\lambda_1 = 1$! (This is the equilibrium state.)
2. Find the eigenvalue λ_2 of T corresponding to the eigenvector $\bar{v}_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$!
3. Calculate α and β in $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha\bar{v}_1 + \beta\bar{v}_2$!
4. Calculate $T(\alpha\bar{v}_1 + \beta\bar{v}_2)$, $T^2(\alpha\bar{v}_1 + \beta\bar{v}_2)$, etc.