Zh2, Diff.Egy., 2013.05.08. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. (1+2+3+4 pont)
Keresd meg u-t es v-t, ha $f(z) = f(x+iy) = u(x,y) + iv(x,y)$, ahol $f = z^2 - \bar{z}$!

$$f = (x+iy)^2 - (x-iy) = \underbrace{(x^2-y^2-x)}_{U} + i\underbrace{(2xy+y)}_{V}$$

Ird fel es ellenorizd a CR egyenleteket! Differencialhato-e f?

$$\frac{\partial}{\partial x}(U+iV) = \frac{1}{i}\frac{\partial}{\partial y}(U+iV) \qquad U_x = V_y \qquad V_x = -U_y$$

$$(x^2-y^2-x)'_x \neq (2xy+y)'_y \qquad (2xy+y)'_x = -(x^2-y^2-x)'_y$$

$$2x-1 \neq 2x+1 \qquad 2y = -(-2y) \qquad \text{NEM}$$

Szamold ki a definicio alapjan a kovetkezo integralt!

$$\oint_\Gamma \frac{1}{z^5}\,dz, \qquad \Gamma = \{z(t) = \underbrace{3\cos t + 3i\sin t}_{3\cdot e^{it}},\ 0 \le t \le 2\pi\}$$

$$\oint_\Gamma \frac{1}{z^5}\,dz = \int_0^{2\pi} (3\cdot e^{it})^{-5}\cdot d(3e^{it}) = \int_0^{2\pi} \frac{1}{3^5}\cdot e^{-5it}\cdot 3\cdot i\cdot e^{it}\,dt$$

$$= \frac{1}{3^4}\int_0^{2\pi} e^{-4it}\,dt = \frac{1}{3^4}\left[\frac{e^{-4it}}{-4i}\right]_{t=0}^{2\pi} = 0$$

Szamold ki a kovetkezo integralt!

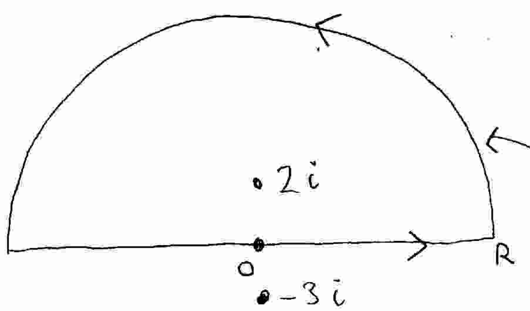
$$\int_{-\infty}^{\infty} \frac{\cancel{(3+2i)x-(6+6i)}\ -10}{x^2+ix+6}\,dx. =$$

$$= \int_{-\infty}^{\infty} \frac{2i}{z-2i}\,dz + \int_{-\infty}^{\infty} \frac{-2i}{z+3i}\,dz \approx \int_{\frown 0} \frac{2i}{z-2i}\,dz + \int_{\frown 0} \frac{-2i}{z+3i}\,dz =$$

$$= 2i \cdot 2\pi i + 0 = -4\pi$$

a $2i$ pólushely bélülvan a fikörön

a $-3i$ pólushely kivül van a félkörön



$$\left|\int_{\frown} \frac{-10}{z^2+iz+6}\,dz\right| \text{"}\le\text{"} R\pi \cdot \frac{-10}{R^2} \longrightarrow 0$$

ha $R \to \infty$

$\hookrightarrow$ A2 $\frown$ iv hossza

2. (1+1+1+5+2 pont)
Keresd meg a kovetkezo DE altalanos megoldasat! $y' = \delta(x)$.

$$y(x) = H(x) + C$$

Keresd meg a kovetkezo DE megoldasat az ($y(x) = 0$, ha $x < 0$) feltetel mellett! $y'' = \delta(x)$.

$$y'(x) = H(x)$$

$$y(x) = \begin{cases} 0, & \text{ha } x < 0 \\ x, & \text{ha } x > 0 \end{cases}$$

Keresd meg a kovetkezo DE megoldasat a ($G(x) = 0$, ha $x < 0$) feltetel mellett!
$G'' + 4G' + 13G = \delta(x)$.

Add meg a kovetkezo mennyisegeket!

$G(0^+) - G(0^-) =$ 0 $G'(0^+) - G'(0^-) =$ 1

Mennyi $G(x)$? Ha $x < 0$ $G(x) = 0$

Ha $x > 0$ $G(x) = y(x)$, ahol

$$y'' + 4y' + 13y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

$$\lambda^2 + 4\lambda + 13 = 0 \longrightarrow \lambda_{1,2} = -2 \pm 3i$$

$$y(x) = e^{-2x}(C_1 \cos(3x) + C_2 \sin(3x))$$

$$y'(x) = -2e^{-2x}(C_1 \cos(3x) + C_2 \sin(3x)) + e^{-2x}(-3C_1 \sin(3x) + 3C_2 \cos(3x))$$

$$y(0) = 0 \longrightarrow C_1 = 0 \qquad y'(0) = 1 \longrightarrow C_2 = \frac{1}{3}$$

$$G(x) = \begin{cases} 0, & \text{ha } x < 0 \\ \frac{1}{3} e^{-2x} \cdot \sin(3x), & \text{ha } x > 0 \end{cases}$$

Ird fel az $y'' + 4y' + 13y = f(x)$ DE megoldasat, ha ($y(x) = f(x) = 0$, ha $x \ll 0$).

$$y(x) = \int_{-\infty}^{\infty} G(x-z) f(z)\, dz =$$

$$= \int_{-\infty}^{x} \frac{1}{3} e^{-2(x-z)} \cdot \sin(3 \cdot (x-z)) \cdot f(z)\, dz$$

3. (2+2+3+2+1 pont)
Szamold ki az $f = tH(t-5)$ fuggveny Laplace transzformaltjat a definicio alapjan!

$$\mathcal{L}(f) = \int_0^\infty e^{-st} \cdot tH(t-5)\,dt = \int_5^\infty e^{-st} \cdot t = \left[-\frac{e^{-st} \cdot t}{s} - \frac{e^{-st}}{s^2} \right]_5^\infty$$

$$= \frac{e^{-s \cdot 5} \cdot 5}{s} + \frac{e^{-s \cdot 5}}{s^2} \quad (\text{ha } \mathrm{Re}(s) > 0)$$

$f(t-\tau) = 1$

Szamold ki az alabbi fuggvenypar $f*g$ konvoluciojat! $f(t) = 1$, $g(t) = \sin 5t$

$$(f*g)(t) = \int_0^t 1 \cdot \sin(5\tau)\,d\tau = \left[\frac{-\cos(5\tau)}{5} \right]_{\tau=0}^t =$$

$$= \left(-\frac{\cos(5t)}{5} \right) - \left(-\frac{\cos(0)}{5} \right) = -\frac{\cos(5t)}{5} + \frac{1}{5}$$

Legyen $y'' + 4y' + 13y = (1+t)^3$, $y(0) = 5$, $y'(0) = 7$. Mennyi $\mathcal{L}(y(t)) = Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$.)

$$\left(s^2 Y(s) - 5s - 7 \right) + 4\left(sY(s) - 5 \right) + 13\,Y(s) = \frac{3!}{s^4} + 3 \cdot \frac{2!}{s^3} + 3\frac{1}{s^2} + \frac{1}{s}$$

$$Y(s) = \frac{1}{s^2+4s+13} \left(5s + 27 + \frac{6}{s^4} + \frac{6}{s^3} + \frac{3}{s^2} + \frac{1}{s} \right)$$

Ird fel azt, hogy hogyan nez ki $Y(s)$ parcialis tort felbontasa! (Az együtthatókat nem kell kiszámolni!)

$$s^2+4s+13 = (s+2+3i)(s+2-3i)$$

$$Y(s) = \frac{A}{s+2+3i} + \frac{B}{s+2-3i} + \frac{C}{s^4} + \frac{D}{s^3} + \frac{E}{s^2} + \frac{F}{s}$$

Mennyi $y(t)$? ($\mathcal{L}(e^{at}) = \frac{1}{s-a}$.)

$$y(t) = A\,e^{(-2-3i)t} + B\,e^{(-2+3i)t} + \frac{C}{3!}t^3 + \frac{D}{2!}t^2 + E \cdot t + F$$

4. (3+2+2+3 pont)
Oldd meg az $y'' = \lambda y$ DE-t az $y(0) = 0,\ y'(L) = 0$ feltetelek mellett y-ra es λ-ra!

$$y'' = \lambda y \longrightarrow y = C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}, \quad y' = C_1\sqrt{\lambda}e^{\sqrt{\lambda}x} - C_2\sqrt{\lambda}e^{-\sqrt{\lambda}x}$$

$$y(0) = 0 \longrightarrow C_1 = -C_2, \quad y'(L) = 0 = C_1\sqrt{\lambda}e^{\sqrt{\lambda}L} + C_1\sqrt{\lambda}e^{-\sqrt{\lambda}L} \longrightarrow$$

$$\longrightarrow e^{\sqrt{\lambda}L} = -e^{-\sqrt{\lambda}L} \longrightarrow e^{2\sqrt{\lambda}L} = -1 \longrightarrow 2\sqrt{\lambda}L = (2k+1)\cdot i\cdot\pi \longrightarrow$$

$$\longrightarrow \lambda_k = -\left[\left(k+\frac{1}{2}\right)\frac{\pi}{L}\right]^2, \quad y_k = \sin\left[\left(k+\frac{1}{2}\right)\cdot\frac{\pi}{L}\right]\cdot\frac{\sqrt{2}}{\sqrt{L}}, \ \text{igy} \int_0^L |y_k|^2 dx = 1$$

Legyen $f(x) = x = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, ha $x \in (-\pi,\pi)$ Mennyi $\hat{f}_3$?

$$\hat{f}_3 = \left(\frac{e^{i3x}}{\sqrt{2\pi}}, x\right) = \int_{-\pi}^{\pi} \overline{\frac{e^{i3x}}{\sqrt{2\pi}}}\cdot x\,dx = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} e^{-i3x}\cdot x\,dx =$$

$$= \frac{1}{\sqrt{2\pi}}\left[\frac{e^{-i3x}}{-i\cdot 3}x - \frac{e^{-i3x}}{(-i3)^2}\right]_{-\pi}^{\pi} = -i\frac{\sqrt{2\pi}}{3}$$

$y_t(t,x) = y_{xx}(t,x),\ y(0,x) = \cos(7x)$. Mennyi $y(t,x)$?

$$y(0,x) = \frac{\sqrt{2\pi}}{2}\frac{e^{i7x}}{\sqrt{2\pi}} + \frac{\sqrt{2\pi}}{2}\cdot\frac{e^{-i7x}}{\sqrt{2\pi}}$$

$$y(t,x) = \frac{\sqrt{2\pi}}{2}\frac{e^{i7x}}{\sqrt{2\pi}}\cdot e^{-7^2t} + \frac{\sqrt{2\pi}}{2}\cdot\frac{e^{-i7x}}{\sqrt{2\pi}}\cdot e^{-(-7)^2t}$$

$$= \cos(7x)\cdot e^{-49t}$$

$y_{tt}(t,x) = y_{xx}(t,x),\ y(0,x) = \cos(7x),\ y'(0,x) = \sin(5x)$. Mennyi $y(t,x)$?

$$y(0,x) = \frac{\sqrt{2\pi}}{2}\frac{e^{i7x}}{\sqrt{2\pi}} + \frac{\sqrt{2\pi}}{2}\frac{e^{-i7x}}{\sqrt{2\pi}}$$

$$y_t'(0,x) = \frac{\sqrt{2\pi}}{2i}\frac{e^{i5x}}{\sqrt{2\pi}} - \frac{\sqrt{2\pi}}{2i}\frac{e^{-i5x}}{\sqrt{2\pi}}$$

$$y(t,x) = \frac{\sqrt{2\pi}}{2}\frac{e^{i7x}}{\sqrt{2\pi}}\cdot\cos(7t) + \frac{\sqrt{2\pi}}{2}\frac{e^{-i7x}}{\sqrt{2\pi}}\cdot\cos((-7)t) +$$

$$+ \frac{\sqrt{2\pi}}{2i}\frac{e^{i5x}}{\sqrt{2\pi}}\frac{\sin(5t)}{5} - \frac{\sqrt{2\pi}}{2i}\frac{e^{-i5x}}{\sqrt{2\pi}}\cdot\frac{\sin(-5t)}{-5}$$

$$= \cos(7x)\cdot\cos(7t) + \sin(5x)\cdot\frac{\sin(5t)}{5}$$