Zh1, Diff.Egy., 2013.04.02. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. (3+4+3 pont)
a)

$$y' = f(x,y) = 2y + yx;$$

Mennyi y'' ? Ird fel y masodrendu Talor polinomjat az $x = 0$ pont korul, ha $y(0) = 3$!

$y(0) = 3$

$y'(0) = 2\cdot 3 + 3\cdot 0 = 6$

$$\boxed{y(x) \approx T_2(x) = 3 + 6x + \frac{15}{2!}x^2}$$

$$y''(x) = \frac{d}{dx}y'(x) = \left(\frac{\partial}{\partial x} + (2y+yx)\frac{\partial}{\partial y}\right)(2y+yx)$$

$$= y + (2y+yx)(2+x), \text{ tehát } y''(0) = 3 + (2\cdot 3 + 3\cdot 0)(2+0) = 15$$

b) Alkalmazd az Euler, illetve a Heun modszert a kovetkezo DE-re $\Delta x = 0.1$ lepeskozzel az $y(2) = 3$ kezdeti feltetel mellett!

$$y' = x^2 - y^3;$$

~~Mit josol a ket modszer $y(2.1)$-re?~~

Euler:

$$y(2.1) \approx y(2) + y'(2,3)\cdot 0.1 = 3 + (2^2 - 3^3)\cdot 0.1 = 3 + (-2.3) = 0.7$$

Heun: $k_1 = y'(2,3) = -23, \quad k_2 = y'(2+0.1;\ 3 + (-23)\cdot 0.1) = 2.1^2 - 0.7^3$

$$y(2.1) \approx 3 + \frac{1}{2}\left((-23)^2 + (2.1^2 - 0.7^3)\right)\cdot 0.1$$

c) Legyen $f(x) = \cancel{\sqrt[3]{27}}$ $\sqrt[3]{x}$, $x_0 = \cancel{x}$ 27. Ird fel f-nek a linearis $f(x_0 + \Delta x) \approx T_1(x_0 + \Delta x)$ kozeliteset, ha $\Delta x = 0.1$!
Mennyi $\max_{z\in[x_0,x_0+\Delta x]}|f''(z)|$? Adj nemtrivialis felso korlatot a kozelites
$|\text{hiba}(\Delta x)| = |f(x_0 + \Delta x) - T_1(x_0 + \Delta x)|$ hibajara!

$f(27) = \sqrt[3]{27} = 3$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(27) = \frac{1}{3}\frac{1}{\sqrt[3]{27^2}} = \frac{1}{27}$

$f(27.1) \approx T_1(27.1) = 3 + \frac{1}{27}\cdot 0.1$

$f''(x) = \frac{1}{3}\cdot\left(-\frac{2}{3}\right)x^{-5/3}$

$$\max_{z\in[27,27.1]}\left|-\frac{2}{9}z^{-5/3}\right| = \frac{2}{9}\cdot 27^{-5/3} = \frac{2}{3^7}$$

hiba:

$$|f(27.1) - T_1(27.1)| \leq \frac{1}{2}\cdot\frac{2}{3^7}\cdot 0.1^2$$

2. (5+2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -y_1 \\ y_1 - 2y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Keresd meg A sajatertekeit es sajatvektorait!

$$A = \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix} \qquad \begin{vmatrix} -1-\lambda & 0 \\ 1 & -2-\lambda \end{vmatrix} = 0 = (-1-\lambda)(-2-\lambda) - 0 \cdot 1$$

$$\lambda_1 = -1 \qquad \lambda_2 = -2$$

$$\begin{pmatrix} -1-(-1) & 0 \\ 1 & -2-(-1) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} -1-(-2) & 0 \\ 1 & -2-(-2) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x = y \qquad x = 0$$

$$V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad V_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Ird fel a DE altalanos megoldasat!

$$y_{ált} = C_1 e^{-1 \cdot x} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{-2 \cdot x} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Szamold ki a DE partikularis megoldasat!

$$C_1 \cdot e^{-1 \cdot 0} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{-2 \cdot 0} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$C_1 = 1 \qquad C_2 = 2$$

$$y_{part} = 1 \cdot e^{-1 \cdot x} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 2 \cdot e^{-2x} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(5 × 2 pont)
Ird fel, hogy milyen osszefugges van A es a sajatertekeket tartalmazo diagonalis D matrixok kozott!

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix} \Big(\underbrace{\boxed{\begin{matrix} 1 \\ 1 \end{matrix}}}_{v_1} \underbrace{\boxed{\begin{matrix} 0 \\ 1 \end{matrix}}}_{v_2} \Big) = \begin{pmatrix} \underbrace{\boxed{-1}}_{\lambda_1} & 0 \\ 0 & \underbrace{\boxed{-2}}_{\lambda_2} \end{pmatrix}$$

$D = S^{-1} A S$

vagy

$A = S D S^{-1}$

Mennyi e^{xA}? (Elegendo az, hogy kifejezed az eredmenyt D, illetve egy S matrix es annak inverze szorzatakent!)

$$e^{xA} = S e^{xD} S^{-1} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{-1 \cdot x} & 0 \\ 0 & e^{-2x} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{-1}$$

Ird fel a partikularis megoldast e^{xA} segitsegevel!

$$y_{part} = e^{xA} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

2b) Ird at a kovetkezo DE rendszert elsorendu DE rendszerre!

$$\frac{d^2}{dx^2} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4y_1' - y_2^2 \\ y_2' - y_1' \end{pmatrix}$$

$$\frac{d}{dx} \begin{pmatrix} y_1 \\ v_1 \\ y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ 4v_1 - y_2^2 \\ v_2 \\ v_2 - v_1 \end{pmatrix}$$

Ird at a kovetkezo DE-ket idofuggetlen DE rendszerre!

$$\frac{d}{dx} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x + y_2 \\ y_1 - y_2 + x^5 \end{pmatrix}$$

$$\frac{d}{dx} \begin{pmatrix} y_1 \\ y_2 \\ t \end{pmatrix} = \begin{pmatrix} t + y_2 \\ y_1 - y_2 + t^5 \\ 1 \end{pmatrix}$$

Jegy:

1	0-15
2	16-22
3	23-28
4	29-34
5	35-40

3a. (1+1+1+2 pont)

$y' = 1 - y^6. = f(y)$

Keresd meg a DE fixpöntjait!

$$f(y) = 0 = 1 - y^6 \longrightarrow \quad y_1 = -1 \qquad y_2 = 1$$

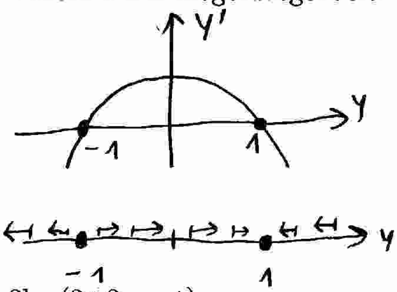
Ird fel a fixpontok koruli linearizalt kozelito DE-t!

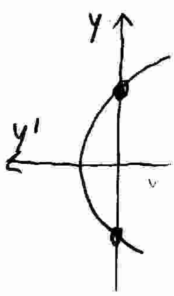
$$f' = \frac{df(y)}{dy} = -6y^5 \quad \Bigg| \quad f'(-1) = 6, \quad \frac{d}{dx}(y-(-1)) = \frac{d}{dx}\Delta y_1 = 6\Delta y_1 \quad \Bigg| \quad f'(1) = -6, \quad \frac{d}{dx}(y-1) = \frac{d}{dx}\Delta y_2 = -6\Delta y_2$$

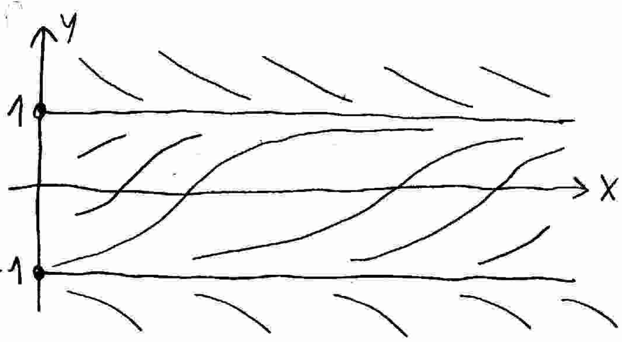
Ha $y(0) = 0$, mennyi

$\lim_{x\to\infty} y(x) =$ 1 $\qquad \lim_{x\to-\infty} y(x) =$ -1

Vazold a DE megoldasgorbeit!







3b. (2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} (y_2^3 + 8)(y_1 - 3) \\ (y_1^3 - 1)y_2. \end{pmatrix}$$

Keresd meg a DE fixpontjat!

$$\left.\begin{aligned} (y_2^3+8)(y_1-3) &= 0 \\ (y_1^3-1)y_2 &= 0 \end{aligned}\right\} \Rightarrow \quad y_2 = -2,\ y_1 = 1 \qquad P_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

vagy

$$y_1 = 3,\ y_2 = 0 \qquad P_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

Ird fel a fixpont koruli linearizalt kozelito DE-t!

Jacobi mátrix:

$$J = \begin{pmatrix} \frac{\partial}{\partial y_1}(y_2^3+8)(y_1-3) & \frac{\partial}{\partial y_2}(y_2^3+8)(y_1-3) \\ \frac{\partial}{\partial y_1}(y_1^3-1)y_2 & \frac{\partial}{\partial y_2}(y_1^3-1)y_2 \end{pmatrix} = \begin{pmatrix} y_2^3+8 & 3y_2^2(y_1-3) \\ 3y_1^2 y_2 & (y_1^3-1) \end{pmatrix}$$

$$J(P_1) = \begin{pmatrix} 0 & -24 \\ -6 & 0 \end{pmatrix} \qquad\Bigg|\qquad J(P_2) = \begin{pmatrix} 8 & 0 \\ 0 & 26 \end{pmatrix}$$

$$\frac{d}{dx}\begin{pmatrix} y_1-1 \\ y_2-(-2) \end{pmatrix} = \frac{d}{dx}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix} = \begin{pmatrix} 0 & -24 \\ -6 & 0 \end{pmatrix}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix} \qquad\Bigg|\qquad \frac{d}{dx}\begin{pmatrix} y_1-3 \\ y_2-0 \end{pmatrix} = \frac{d}{dx}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 26 \end{pmatrix}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix}$$