Pót 05.15
Zh1, Diff.Egy., 2013.~~04.02.~~ NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. (4+(3+3) pont)
a)

$$y' = f(x, y) = x^2 + 5y - 2;$$

Mennyi y'' ? Ird fel y masodrendu Talor polinomjat az $x = 0$ pont korul, ha $y(0) = 3$!

$$y'' = \left(f(x,y)\frac{\partial}{\partial y} + \frac{\partial}{\partial x}\right)(x^2+5y-2) = (x^2+5y-2)(5) + 2x$$

$$y'(0) = 0^2 + 5\cdot 3 - 2 = 13 \qquad y''(0) = (0^2 + 5\cdot 3 - 2)\cdot 5 + 2\cdot 0 = 65$$

$$y(x) \approx T_2(x) = 3 + 13x + \frac{65}{2!}x^2$$

b) Alkalmazd az Euler, illetve a Heun modszert a kovetkezo DE-re $\Delta x = 0.01$ lepeskozzel! az $\bar{y}(2) =$ kezdeti feltetel mellett!

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} y_1 - y_2 \\ 3y_2^2 + x. \end{pmatrix}, \qquad \bar{y}(2) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Mit josol a ket modszer $\bar{y}(2.01)$-re?
Euler:

$$\bar{y}(2.01) \approx \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 0.01\cdot\begin{pmatrix} 2-3 \\ 3\cdot 3^2 + 2 \end{pmatrix} = \begin{pmatrix} 1.99 \\ 3.29 \end{pmatrix}$$

Heun:

$$\bar{y}(2.01) \approx \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \frac{1}{2}\left[\begin{pmatrix} 2-3 \\ 3\cdot 3^2 + 2 \end{pmatrix} + \begin{pmatrix} 1.99 - 3.29 \\ 3\cdot 3.29^2 + 2.01 \end{pmatrix}\right]$$

2. (5+2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -y_1 - 3y_2 \\ 3y_1 - y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Keresd meg A sajaterteket es sajatvektorait!

$$0 = \det(A - \lambda E) = \begin{vmatrix} -1-\lambda & -3 \\ 3 & -1-\lambda \end{vmatrix} = (-1-\lambda)^2 - (-3)\cdot 3 = \lambda^2 + 2\lambda + 10$$

$$\lambda_1 = -1+3i \qquad \lambda_2 = -1-3i$$

① $\begin{pmatrix} -1 & -3 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = (-1+3i)\begin{pmatrix} x \\ y \end{pmatrix}$,

vagy

$$\begin{pmatrix} -1-(-1+3i) & 3 \\ -3 & -1-(-1+3i) \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{matrix} -3ix + 3y = 0 \\ -3x - 3iy = 0 \end{matrix}\right\} \rightarrow y = ix \qquad V_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}$$

②

$$\begin{pmatrix} -1-(-1-3i) & 3 \\ -3 & -1-(-1-3i) \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{matrix} 3ix + 3y = 0 \\ -3x + 3iy = 0 \end{matrix}\right\} \rightarrow y = -ix \qquad V_2 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Ird fel a DE altalanos megoldasat!

$$\overline{y}_{ált}(x) = C_1 e^{(-1+3i)x}\begin{pmatrix} 1 \\ i \end{pmatrix} + C_2 e^{(-1-3i)x}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Szamold ki a DE partikularis megoldasat!

$$C_1\begin{pmatrix} 1 \\ i \end{pmatrix} + C_2\begin{pmatrix} 1 \\ -i \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \Rightarrow \begin{matrix} c_1 + c_2 = 1 \\ ic_1 - ic_2 = 3 \\ \hline c_1 - c_2 = -3i \end{matrix}$$

$$\Downarrow$$

$$c_1 = \frac{1-3i}{2}, \quad c_2 = \frac{1+3i}{2}$$

$$\overline{y}_{part}(x) = \frac{1-3i}{2} e^{(-1+3i)x}\begin{pmatrix} 1 \\ i \end{pmatrix} + \frac{1+3i}{2} e^{(-1-3i)x}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

(5 × 2 pont)

$$A = \begin{pmatrix} 5 & 0 \\ 6 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 6x & 0 \end{pmatrix} + \frac{1}{2!} \underbrace{\begin{pmatrix} 0 & 0 \\ 6x & 0 \end{pmatrix}^2}_{0} +$$

Mennyi e^{xA}?

$$\exp(xA) = \exp\left[x \cdot \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 6x & 0 \end{pmatrix}\right] = \exp\left[x \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\right] \cdot \exp\begin{pmatrix} 0 & 0 \\ 6x & 0 \end{pmatrix}$$

$$= \begin{pmatrix} e^{5x} & 0 \\ 0 & e^{5x} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 6x & 1 \end{pmatrix} = \begin{pmatrix} e^{5x} & 0 \\ 6xe^{5x} & e^{5x} \end{pmatrix}$$

Ird fel a

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 5y_1 \\ 6y_1 + 5y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

DE partikularis megoldasat e^{xA} segitsegevel!

$$\bar{y}(x) = e^{xA}\,\bar{y}(0) = \begin{pmatrix} e^{5x} & 0 \\ 6xe^{5x} & e^{5x} \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

2b) Ird at a kovetkezo DE rendszert elsorendu DE rendszerre!

$$\frac{d^2}{dx^2} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_1'^2 - y_2 \\ 2y_2' - 3y_1' \end{pmatrix}$$

$$\frac{d}{dx} \begin{pmatrix} y_1 \\ v_1 \\ y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_1^2 - y_2 \\ v_2 \\ 2v_2 - 3v_1 \end{pmatrix}$$

Ird at a kovetkezo DE-ket idofuggetlen DE rendszerre!

$$\frac{d}{dx} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x + x^5 \\ y_1 y_2 + x^5 \end{pmatrix}$$

$$\frac{d}{dx} \begin{pmatrix} y_1 \\ y_2 \\ t \end{pmatrix} = \begin{pmatrix} t + t^5 \\ y_1 y_2 + t^5 \\ 1 \end{pmatrix}$$

3a. (1+1+1+2 pont)

$y' = (y^2-4)y. = y^3 - 4y$

Keresd meg a DE fixpontjait!

$$(y^2-4)y = (y+2)(y-2)y = 0 \rightarrow y_1 = -2,\ y_2 = 2,\ y_3 = 0$$

Ird fel a fixpontok koruli linearizalt kozelito DE-t! $\frac{d}{dy}(y^3-4y) = 3y^2-4$

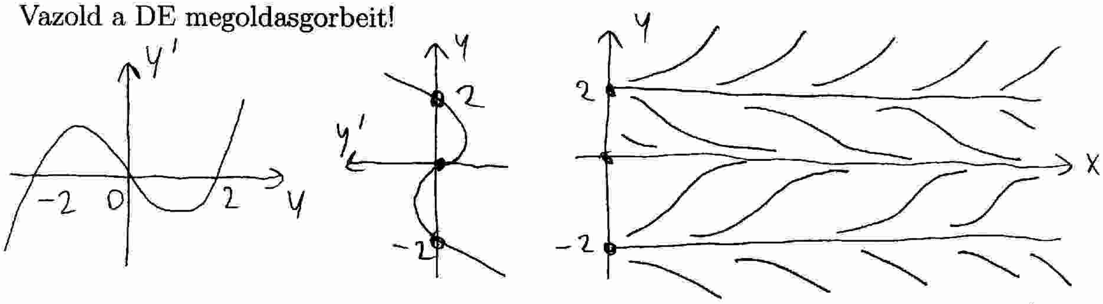
$$\frac{d}{dx}(y-(-2)) = \frac{d}{dx}\Delta y_1 = \underset{\uparrow \atop 3\cdot(-2)^2-4}{8}\Delta y_1,\quad \frac{d}{dx}(y-2) = \frac{d}{dx}\Delta y_2 = \underset{\uparrow \atop 3\cdot 2^2-4}{8}\Delta y_2,\quad \frac{d}{dx}(y-0) = \frac{d}{dx}\Delta y_3 = \underset{\nearrow \atop 3\cdot 0^2-4}{-4}\Delta y_3$$

Ha $y(0) = 0$, mennyi

$\lim_{x\to\infty} y(x) = 0$ $\qquad$ $\lim_{x\to-\infty} y(x) = 0$

Mivel $y=0$ fixpont (egyensúlyi állapot)

Vazold a DE megoldasgorbeit!



3b. (2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} y_1 - y_2 + 1 \\ (3y_1 - 2y_2)(5y_1 - 4y_2). \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

Keresd meg a DE fixpontjait!

$y_1 - y_2 + 1 = 0$

$(3y_1 - 2y_2)(5y_1 - 4y_2) = 0$

$y_2 = y_1 + 1$, tehát

$(3y_1 - 2(y_1+1))$, vagy $(5y_1 - 4(y_1+1)) = 0$

$$\Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} = P_1 \text{ vagy } \begin{pmatrix} 4 \\ 5 \end{pmatrix} = P_2$$

Ird fel a fixpont koruli linearizalt kozelito DE-t!

$$\mathrm{Jac} = \begin{pmatrix} \frac{\partial f_1}{\partial y_1} & \frac{\partial f_1}{\partial y_2} \\ \frac{\partial f_2}{\partial y_1} & \frac{\partial f_2}{\partial y_2} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 3(5y_1-4y_2) + (3y_1-2y_2)\cdot 5 & * \end{pmatrix}$$

$(-2)\cdot(5y_1-4y_2) + (3y_1-2y_2)\cdot(-4)$

$$\mathrm{Jac}(P_1) = \begin{pmatrix} 1 & -1 \\ -6 & 4 \end{pmatrix} \qquad \mathrm{Jac}(P_2) = \begin{pmatrix} 1 & -1 \\ 10 & -8 \end{pmatrix}$$

$$\frac{d}{dx}\begin{pmatrix} y_1 - 2 \\ y_2 - 3 \end{pmatrix} = \frac{d}{dx}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix} \approx \begin{pmatrix} 1 & -1 \\ -6 & 4 \end{pmatrix}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix} \quad \Bigg| \quad \frac{d}{dx}\begin{pmatrix} y_1 - 4 \\ y_2 - 5 \end{pmatrix} = \frac{d}{dx}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix} \approx \begin{pmatrix} 1 & -1 \\ 10 & -8 \end{pmatrix}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix}$$

PotZh2, Diff.Egy., 2013.05.15. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. (1+2+3+4 pont)

Keresd meg u-t es v-t, ha $f(z) = f(x+iy) = u(x,y) + iv(x,y)$, ahol $f = e^{3z-\bar{z}}$!

$$f = e^{3(x+iy)-(x-iy)} = e^{2x+4iy} = e^{2x}\left(\cos(4y) + i\sin(4y)\right) =$$
$$= \left[e^{2x}\cos(4y)\right] + i\left[e^{2x}\sin(4y)\right] = [u] + i[v]$$

Ird fel es ellenorizd a CR egyenleteket! Differencialhato-e f? $\frac{\partial}{\partial x}(u+iv) = \frac{1}{i}\frac{\partial}{\partial y}(u+iv)$

$$u_x \neq v_y$$
$$(e^{2x}\cos 4y)'_x \neq (e^{2x}\sin 4y)'_y$$
$$2e^{2x}\cos 4y \neq e^{2x}\cdot 4\cdot\cos 4y$$

$$v_x \neq -u_y$$
$$(e^{2x}\sin 4y)'_x \neq -(e^{2x}\cos 4y)'_y$$
$$2e^{2x}\sin 4y \neq -e^{2x}\cdot(-4\sin 4y)$$

NEM diff.

Szamold ki a definicio alapjan a kovetkezo integralt!

$$\int_\Gamma \frac{1}{z^2}dz \qquad \cancel{\oint_\Gamma \frac{1}{z^2}\,dz}, \qquad \Gamma = \{z(t) = \overbrace{3\cos t + 3i\sin t}^{3\cdot e^{it}},\ 0 \le t \le \pi\}$$

$$\frac{dz}{dt} = 3ie^{it} \longrightarrow dz = 3ie^{it}dt$$

$$\int_\Gamma \frac{1}{z^2}dz = \int_{t=0}^{\pi} \frac{1}{(3e^{it})^2}\cdot 3ie^{it}dt = \frac{i}{3}\int_{t=0}^{\pi} e^{-it}dt = \frac{i}{3}\left[\frac{e^{-it}}{-i}\right]_{t=0}^{\pi}$$
$$= -\frac{1}{3}\left(e^{-i\pi} - e^{0}\right) = -\frac{1}{3}(-1-1) = \frac{2}{3}$$

Szamold ki a kovetkezo integralt!

$$\int_{-\infty}^{\infty} \frac{2+2i}{x^2-2i}\,dx =$$

$$= \oint_{⌓} \frac{2+2i}{z^2-2i} = \oint_{⌓} \frac{A}{z-(1+i)} + \frac{B}{z-(1-i)}\,dz$$

$$= \underbrace{\oint_{⌓} \frac{1}{z-(1+i)}dz}_{=1\cdot 2\pi i} + \underbrace{\oint_{⌓} \frac{1}{z-(-1-i)}dz}_{\text{nulla, mivel a } z=(-1-i) \text{ pont kívül van a félkörön}} =$$

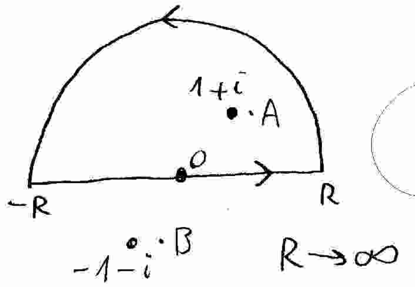
$$= 1\cdot 2\pi i$$

$$z^2 - 2i = 0 \longrightarrow z_1 = 1+i, \quad z_2 = -1-i$$

$$A(z-(-1-i)) + B(z-(1+i)) = 2+2i$$

$$\longrightarrow A = -B,$$

$$A = 1, \quad B = 1$$



2. (1+1+1+5+2 pont)

Keresd meg a kovetkezo DE altalanos megoldasat! $y' = \frac{1}{3}\delta(x) + 1$.

$$y(x) = \frac{1}{3} H(x) + x + C$$

Keresd meg a kovetkezo DE megoldasat az ($y(x) = 0$, ha $x < 0$) feltetel mellett! $2y'' = \delta(x)$.

$y'' = \frac{1}{2}\delta$

$y' = \frac{1}{2} H$

$$y(x) = \begin{cases} 0, & \text{ha } x < 0 \\ \frac{1}{2}x, & \text{ha } x > 0 \end{cases}$$

Megjegyzés: $y(0^+) - y(0^-) = 0$

$y'(0^+) - y'(0^-) = \frac{1}{2}$

Ezeket az adatokat kell majd ITT használni!

Keresd meg a kovetkezo DE megoldasat a ($G(x) = 0$, ha $x < 0$) feltetel mellett!
$2G'' + 10G' + 12G = \delta(x)$.

Add meg a kovetkezo mennyisegeket!

$G(0^+) - G(0^-) =$ 0 $\qquad$ $G'(0^+) - G'(0^-) =$ $\frac{1}{2}$

Mennyi $G(x)$? Ha $x < 0$, $G(x) = 0$

Ha $x > 0$, $G(x) = y(x)$, ahol $y(x)$ megoldása az

$2y'' + 10y' + 12y = 0$, $y(0) = 0$, $y'(0) = \frac{1}{2}$ DE-nek.

$$2\lambda^2 + 10\lambda + 12 = 0 \longrightarrow \lambda_1 = -2,\ \lambda_2 = -3$$

$$y = C_1 e^{-2x} + C_2 e^{-3x}, \quad C_1 + C_2 = 0, \quad -2C_1 - 3C_2 = \frac{1}{2}$$

$$C_1 = \frac{1}{2}, \quad C_2 = -\frac{1}{2}$$

$$G(x) = \begin{cases} 0, & \text{ha } x < 0 \\ \frac{1}{2}e^{-2x} - \frac{1}{2}e^{-3x}, & \text{ha } x > 0 \end{cases}$$

$2y'' + 10y' + 12y = f(x)$ *

Ird fel az ~~$y'' + 4y' + 13y$~~ $= f(x)$ DE megoldasat, ha ($y(x) = f(x) = 0$, ha $x \ll 0$).

$$y(x) = \int_{-\infty}^{\infty} G(x-z) f(z)\, dz =$$

$$= \int_{-\infty}^{x} \left(\frac{1}{2} e^{-2(x-z)} - \frac{1}{2} e^{-3(x+z)} \right) f(z)\, dz$$

*: Ez bizonyos értelemben sajtóhiba, itt elfogadható megoldásnak számít az előző pontban megkapott G használata, vagy az eredeti példa értelmes megoldási kísérlete.

1+1

3. (2+[redacted]+3+2+1 pont)

Szamold ki az $f = e^{6t}H(t-5)$ fuggveny Laplace transzformaltjat a definicio alapjan!

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} \cdot e^{6t} \cdot H(t-5)\,dt = \int_5^\infty e^{(-s+6)t}\,dt = \left[\frac{e^{(-s+6)t}}{-s+6}\right]_5^\infty$$

$$= 0 - \frac{e^{(-s+6)\cdot 5}}{-s+6} = \frac{e^{5(6-s)}}{s-6} \qquad (\text{ha } \operatorname{Re} s > 6)$$

Szamold ki az alabbi fuggvenypar $f*g$ konvoluciojat! $f(t) = e^{6t}$, $g(t) = e^{7t}$

$$(f*g)(t) = \int_0^t e^{6(t-\tau)} \cdot e^{7\tau}\,d\tau = \int_0^t e^{6t+\tau}\,d\tau = \left[e^{6t+\tau}\right]_0^t =$$

$$= e^{7t} - e^{6t}$$

Ez automatikusan nulla!

$$\mathcal{L}(e^{7t} - e^{6t}) = \frac{1}{s-7} - \frac{1}{s-6}$$

$$\mathcal{L}(e^{7t}) \cdot \mathcal{L}(e^{6t}) = \frac{1}{s-7} \cdot \frac{1}{s-6}$$

egyenlőek!

Mennyi $\mathcal{L}(f*g) - \mathcal{L}(f)\mathcal{L}(g)$? =

Legyen $2y'' + 10y' + 12y = e^{-t} + e^{-3t}$, $y(0) = 3$, $y'(0) = 4$. Mennyi $\mathcal{L}(y(t)) = Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$.)

$$2(s^2 Y(s) - 3s - 4) + 10(sY(s) - 3) + 12Y(s) = \frac{1}{s+1} + \frac{1}{s+3}$$

$$Y(s) = \frac{1}{2s^2 + 10s + 12} \cdot \left(6s + 38 + \frac{1}{s+1} + \frac{1}{s+3}\right)$$

$$2s^2 + 10s + 12 = 0$$

$$s_1 = -2, \quad s_2 = -3$$

Ird fel azt, hogy hogyan nez ki $Y(s)$ parcialis tort felbontasa!

$$Y(s) = \frac{\text{polinom}(s)}{(s+2)(s+3)^2(s+1)} = \frac{A}{s+2} + \frac{B}{(s+3)^2} + \frac{C}{s+3} + \frac{D}{s+1}$$

Mennyi $y(t)$? ($\mathcal{L}(e^{at}) = \frac{1}{s-a}$.)

$$y(t) = \mathcal{L}^{-1}(Y(s)) = A e^{-2t} + B^{*} \cdot t \cdot e^{-3t} + C \cdot e^{-3t} + D e^{-1 \cdot t}$$

$$\mathcal{L}(t) = \int_0^\infty e^{-st}\,t\,dt = \frac{1}{s^2}, \quad \mathcal{L}(e^{-3t} \cdot t) = \int_0^\infty e^{-st} \cdot e^{-3t} \cdot t\,dt =$$

$$= \int_0^\infty e^{-(s+3)t} \cdot t\,dt = \frac{1}{(s+3)^2}$$

*A megoldás ezen tag kiszámolása nélkül is helyes

4. (2+3+2+3 pont)

Legyen $f(x) = sgn(x) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, ha $x \in (-\pi, \pi)$ Mennyi $\hat{f}_{-5}$ es ~~X~~?

$$\hat{f}_{-5} = \left(\frac{e^{i(-5)x}}{\sqrt{2\pi}}, sgn(x)\right) = \int_{-\pi}^{\pi} \overline{\frac{e^{-5ix}}{\sqrt{2\pi}}} \cdot sgn\,x\,dx = \int_{-\pi}^{0} \frac{e^{5ix}}{\sqrt{2\pi}} \cdot (-1)\,dx +$$

$$+\int_{0}^{\pi} \frac{e^{5ix}}{\sqrt{2\pi}} \cdot 1\,dx = -\frac{1}{\sqrt{2\pi}}\left[\frac{e^{5ix}}{5i}\right]_{-\pi}^{0} + \frac{1}{\sqrt{2\pi}}\left[\frac{e^{5ix}}{5i}\right]_{0}^{\pi} =$$

$$-\frac{1}{\sqrt{2\pi}\cdot 5i}\left(1 - \underbrace{e^{-5\pi i}}_{-1}\right) + \frac{1}{\sqrt{2\pi}\cdot 5i}\left(\underbrace{e^{+5\pi i}}_{-1} - 1\right) = 4i\frac{1}{\sqrt{2\pi}\cdot 5} = \frac{2\sqrt{2}}{5}\, i\, \frac{1}{\sqrt{\pi}}$$

$y_t'(t,x) = y_{xx}''(t,x)$, $y(0,x) = 8e^{9ix} + \sin x$. Mennyi $y(t,x)$?

Mivel $\frac{\partial^2}{\partial x^2} e^{9ix} = -81 \cdot e^{9ix}$ és $\frac{\partial^2}{\partial x^2}\sin x = -1 \cdot \sin x$, így

$$y(t,x) = 8 \cdot e^{9ix} \cdot e^{-81\cdot t} + \sin x \cdot e^{-1\cdot t}$$

$y_t'(t,x) = y_{xx}''(t,x)$, $y(0,x) = \sum_{n\in\mathbb{Z}} \frac{1}{n}\frac{e^{inx}}{\sqrt{2\pi}}$. Ird fel $y(t,x)$-et vegtelen sor alakban!

$$y(t,x) = \sum_{n\in\mathbb{Z}} \frac{1}{n} \cdot \frac{e^{inx}}{\sqrt{2\pi}} \cdot e^{-n^2 t}$$

$y_{tt}''(t,x) = y_{xx}''(t,x)$, $y(0,x) = 8e^{9ix}$, $y'(0,x) = \cos(5x) + e^{9ix}$. Mennyi $y(t,x)$?

$$y(t,x) = 8 \cdot e^{9ix} \cdot \cos(9t) + \cos(5x) \cdot \frac{\sin(5t)}{5} + e^{9ix} \cdot \frac{\sin(9t)}{9}$$

Pl.: Ha a $y_{tt}'' = y_{xx}''$, $y(0,x) = 0$, $y_t'(0,x) = \cos(5x)$ DE megoldását $y(t,x) = \cos(5x) \cdot c(t)$ alakban keressük, akkor

$y_{xx}'' = -5^2 \cos(5x) \cdot c(t) = y_{tt}'' = \cos(5x) \cdot c''(t)$, vagyis

$-5^2 c(t) = c''(t) \longrightarrow c(t) = C_1 \cos(5t) + C_2 \sin(5t)$.

Továbbá $y(0,x) = 0 \longrightarrow C_1 = 0$

$y_t'(0,x) = \cos(5x) \longrightarrow C_2 = \frac{1}{5}$