4. (3+2+2+3 pont)

Oldd meg az $y'' = \lambda y$ DE-t az $y(0) = 0,\ y'(L) = 0$ feltetelek mellett y-ra es λ-ra!

Legyen $f(x) = x = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, ha $x \in (-\pi, \pi)$ Mennyi $\hat{f}_3$?

$y_t(t,x) = y_{xx}(t,x),\ y(0,x) = \cos(7x)$. Mennyi $y(t,x)$?

$y_{tt}(t,x) = y_{xx}(t,x),\ y(0,x) = \cos(7x),\ y'(0,x) = \sin(5x)$. Mennyi $y(t,x)$?

2. (1+1+1+5+2 pont)

Keresd meg a kovetkezo DE altalanos megoldasat! $y' = \delta(x)$.

Keresd meg a kovetkezo DE megoldasat az ($y(x) = 0$, ha $x < 0$) feltetel mellett! $y'' = \delta(x)$.

Keresd meg a kovetkezo DE megoldasat a ($G(x) = 0$, ha $x < 0$) feltetel mellett!

$G'' + 4G' + 13G = \delta(x)$.

Add meg a kovetkezo mennyisegeket!

$G(0^+) - G(0^-) =$ $\qquad$ $G'(0^+) - G'(0^-) =$

Mennyi $G(x)$?

Ird fel az $y'' + 4y' + 13y = f(x)$ DE megoldasat, ha ($y(x) = f(x) = 0$, ha $x \ll 0$).

3. (2+2+3+2+1 pont)

Szamold ki az $f = tH(t-5)$ fuggveny Laplace transzformaltjat a definicio alapjan!

Szamold ki az alabbi fuggvenypar $f{*}g$ konvoluciojat! $f(t) = 1,\ g(t) = \sin 5t$

Legyen $y'' + 4y' + 13y = (1+t)^3$, $y(0) = 5,\ y'(0) = 7$. Mennyi $\mathcal{L}(y(t)) = Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$.)

Ird fel azt, hogy hogyan nez ki $Y(s)$ parcialis tort felbontasa!

Mennyi $y(t)$? ($\mathcal{L}(e^{at}) = \frac{1}{s-a}$.)

2. (1+1+1+5+2 pont)
Keresd meg a kovetkezo DE altalanos megoldasat! $y' = \delta(x)$.

$$y(x) = H(x) + C$$

Keresd meg a kovetkezo DE megoldasat az ($y(x) = 0$, ha $x < 0$) feltetel mellett! $y'' = \delta(x)$.

$$y'(x) = H(x)$$

$$y(x) = \begin{cases} 0, & \text{ha } x < 0 \\ x, & \text{ha } x > 0 \end{cases}$$

Keresd meg a kovetkezo DE megoldasat a ($G(x) = 0$, ha $x < 0$) feltetel mellett!
$G'' + 4G' + 13G = \delta(x)$.

Add meg a kovetkezo mennyisegeket!
$G(0^+) - G(0^-) =$ 0 $\qquad$ $G'(0^+) - G'(0^-) =$ 1

Mennyi $G(x)$? Ha $x < 0$ $\quad G(x) = 0$

Ha $x > 0 \quad G(x) = y(x)$, ahol

$$y'' + 4y' + 13y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

$$\lambda^2 + 4\lambda + 13 = 0 \longrightarrow \lambda_{1,2} = -2 \pm 3i$$

$$y(x) = e^{-2x}\left(C_1 \cos(3x) + C_2 \sin(3x)\right)$$

$$y'(x) = -2e^{-2x}\left(C_1 \cos(3x) + C_2 \sin(3x)\right) + e^{-2x}\left(-3C_1 \sin(3x) + 3C_2 \cos(3x)\right)$$

$$y(0) = 0 \longrightarrow C_1 = 0 \qquad y'(0) = 1 \longrightarrow C_2 = \frac{1}{3}$$

$$G(x) = \begin{cases} 0, & \text{ha } x < 0 \\ \frac{1}{3} e^{-2x} \cdot \sin(3x), & \text{ha } x > 0 \end{cases}$$

Ird fel az $y'' + 4y' + 13y = f(x)$ DE megoldasat, ha ($y(x) = f(x) = 0$, ha $x \ll 0$).

$$y(x) = \int_{-\infty}^{\infty} G(x-z) f(z)\, dz =$$

$$= \int_{-\infty}^{x} \frac{1}{3} e^{-2(x-z)} \cdot \sin(3 \cdot (x-z)) \cdot f(z)\, dz$$

3. (2+2+3+2+1 pont)

Szamold ki az $f = tH(t-5)$ fuggveny Laplace transzformaltjat a definicio alapjan!

$$\mathcal{L}(f) = \int_0^\infty e^{-st} \cdot t H(t-5)\,dt = \int_5^\infty e^{-st} \cdot t = \left[-\frac{e^{-st} \cdot t}{s} - \frac{e^{-st}}{s^2} \right]_5^\infty$$

$$= \frac{e^{-s \cdot 5} \cdot 5}{s} + \frac{e^{-s \cdot 5}}{s^2} \quad (\text{ha } \operatorname{Re}(s) > 0)$$

$f(t-\tau) = 1$

Szamold ki az alabbi fuggvenypar $f*g$ konvoluciojat! $f(t) = 1$, $g(t) = \sin 5t$

$$(f*g)(t) = \int_0^t 1 \cdot \sin(5\tau)\,d\tau = \left[\frac{-\cos(5\tau)}{5} \right]_{\tau=0}^{t} =$$

$$= \left(-\frac{\cos(5t)}{5} \right) - \left(-\frac{\cos(0)}{5} \right) = -\frac{\cos(5t)}{5} + \frac{1}{5}$$

Legyen $y'' + 4y' + 13y = (1+t)^3$, $y(0) = 5$, $y'(0) = 7$. Mennyi $\mathcal{L}(y(t)) = Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$.)

$$\left(s^2 Y(s) - 5s - 7\right) + 4\left(sY(s) - 5\right) + 13\,Y(s) = \frac{3!}{s^4} + 3 \cdot \frac{2!}{s^3} + 3\frac{1}{s^2} + \frac{1}{s}$$

$$Y(s) = \frac{1}{s^2+4s+13}\left(5s + 27 + \frac{6}{s^4} + \frac{6}{s^3} + \frac{3}{s^2} + \frac{1}{s} \right)$$

Ird fel azt, hogy hogyan nez ki $Y(s)$ parcialis tort felbontasa! (Az együtthatókat nem kell kiszámolni!)

$$s^2+4s+13 = (s+2+3i)(s+2-3i)$$

$$Y(s) = \frac{A}{s+2+3i} + \frac{B}{s+2-3i} + \frac{C}{s^4} + \frac{D}{s^3} + \frac{E}{s^2} + \frac{F}{s}$$

Mennyi $y(t)$? ($\mathcal{L}(e^{at}) = \frac{1}{s-a}$.)

$$y(t) = A e^{(-2-3i)t} + B e^{(-2+3i)t} + \frac{C}{3!}t^3 + \frac{D}{2!}t^2 + E \cdot t + F$$

4. (3+2+2+3 pont)

Oldd meg az $y'' = \lambda y$ DE-t az $y(0) = 0,\ y'(L) = 0$ feltetelek mellett y-ra es λ-ra!

$$y'' = \lambda y \longrightarrow y = C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}, \quad y' = C_1\sqrt{\lambda}\, e^{\sqrt{\lambda}x} - C_2\sqrt{\lambda}\, e^{-\sqrt{\lambda}x}$$

$$y(0) = 0 \longrightarrow C_1 = -C_2, \quad y'(L) = 0 = C_1\sqrt{\lambda}\, e^{\sqrt{\lambda}L} + C_1\sqrt{\lambda}\, e^{-\sqrt{\lambda}L} \longrightarrow$$

$$\longrightarrow e^{\sqrt{\lambda}L} = -e^{-\sqrt{\lambda}L} \longrightarrow e^{2\sqrt{\lambda}L} = -1 \longrightarrow 2\sqrt{\lambda}L = (2k+1)\cdot i \cdot \pi \longrightarrow$$

$$\longrightarrow \lambda_k = -\left[\left(k+\tfrac{1}{2}\right)\frac{\pi}{L}\right]^2, \quad Y_k = \sin\left[\left(k+\tfrac{1}{2}\right)\cdot\frac{\pi}{L}\right]\cdot\frac{\sqrt{2}}{\sqrt{L}}, \text{ igy } \int_0^L |Y_k|^2 dx = 1$$

Legyen $f(x) = x = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, ha $x \in (-\pi, \pi)$ Mennyi $\hat{f}_3$?

$$\hat{f}_3 = \left(\frac{e^{i3x}}{\sqrt{2\pi}}, x\right) = \int_{-\pi}^{\pi} \frac{e^{i3x}}{\sqrt{2\pi}} \cdot x\,dx = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} e^{-i3x}\cdot x\,dx =$$

$$= \frac{1}{\sqrt{2\pi}}\left[\frac{e^{-i3x}}{-i\cdot 3}x - \frac{e^{-i3x}}{(-i3)^2}\right]_{-\pi}^{\pi} = -i\frac{\sqrt{2\pi}}{3}$$

$y_t(t,x) = y_{xx}(t,x),\ y(0,x) = \cos(7x)$. Mennyi $y(t,x)$?

$$y(0,x) = \frac{\sqrt{2\pi}}{2}\frac{e^{i7x}}{\sqrt{2\pi}} + \frac{\sqrt{2\pi}}{2}\cdot\frac{e^{-i7x}}{\sqrt{2\pi}}$$

$$y(t,x) = \frac{\sqrt{2\pi}}{2}\frac{e^{i7x}}{\sqrt{2\pi}}\cdot e^{-7^2 t} + \frac{\sqrt{2\pi}}{2}\cdot\frac{e^{-i7x}}{\sqrt{2\pi}}\cdot e^{-(-7)^2 t}$$

$$= \cos(7x)\cdot e^{-49t}$$

$y_{tt}(t,x) = y_{xx}(t,x),\ y(0,x) = \cos(7x),\ y'(0,x) = \sin(5x)$. Mennyi $y(t,x)$?

$$y(0,x) = \frac{\sqrt{2\pi}}{2}\frac{e^{i7x}}{\sqrt{2\pi}} + \frac{\sqrt{2\pi}}{2}\frac{e^{-i7x}}{\sqrt{2\pi}}$$

$$y_t'(0,x) = \frac{\sqrt{2\pi}}{2i}\frac{e^{i5x}}{\sqrt{2\pi}} - \frac{\sqrt{2\pi}}{2i}\frac{e^{-i5x}}{\sqrt{2\pi}}$$

$$y(t,x) = \frac{\sqrt{2\pi}}{2}\frac{e^{i7x}}{\sqrt{2\pi}}\cdot\cos(7t) + \frac{\sqrt{2\pi}}{2}\frac{e^{-i7x}}{\sqrt{2\pi}}\cdot\cos((-7)t) +$$

$$+ \frac{\sqrt{2\pi}}{2i}\frac{e^{i5x}}{\sqrt{2\pi}}\frac{\sin(5t)}{5} - \frac{\sqrt{2\pi}}{2i}\frac{e^{-i5x}}{\sqrt{2\pi}}\cdot\frac{\sin(-5t)}{-5}$$

$$= \cos(7x)\cdot\cos(7t) + \sin(5x)\cdot\frac{\sin(5t)}{5}$$

4. (3+2+2+3 pont)

Oldd meg az $y'' = \lambda y$ DE-t az $y'(0) = 0,\ y'(L) = 0$ feltetelek mellett y-ra es λ-ra!
Legyen $f(x) = x^2 = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, ha $x \in (-\pi, \pi)$ Mennyi $\hat{f}_3$?
$y_t(t,x) = y_{xx}(t,x),\ y(0,x) = e^{7x} + \sin(x)$. Mennyi $y(t,x)$?
$y_{tt}(t,x) = y_{xx}(t,x),\ y(0,x) = e^{7x} + \sin(x,\ y'(0,x) = \sin(5x) + e^{7x} + \sin(x$. Mennyi $y(t,x)$?

2. (1+1+1+5+2 pont)
Keresd meg a kovetkezo DE altalanos megoldasat! $y' = 3\delta(x)$.
Keresd meg a kovetkezo DE megoldasat az ($y(x) = 0$, ha $x < 0$) feltetel mellett! $4y'' = \delta(x)$.
Keresd meg a kovetkezo DE megoldasat a ($G(x) = 0$, ha $x < 0$) feltetel mellett!
$4G'' + 4G' + 13G = \delta(x)$.
Add meg a kovetkezo mennyisegeket!
$G(0^+) - G(0^-) =$ $G'(0^+) - G'(0^-) =$
Mennyi $G(x)$?
Ird fel az $4y'' + 4y' + 13y = f(x)$ DE megoldasat, ha ($y(x) = f(x) = 0$, ha $x \ll 0$).

3. (2+2+3+2+1 pont)
Szamold ki az $f = \sin t H(t-1)$ fuggveny Laplace transzformaltjat a definicio alapjan!
Szamold ki az alabbi fuggvenypar $f{*}g$ konvoluciojat! $f(t) = t,\ g(t) = e^{5t}$
Legyen $y'' + 16y = t^2$, $y(0) = 5,\ y'(0) = 7$. Mennyi $\mathcal{L}(y(t)) = Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$.)
Ird fel azt, hogy hogyan nez ki $Y(s)$ parcialis tort felbontasa!
Mennyi $y(t)$? ($\mathcal{L}(e^{at}) = \frac{1}{s-a}$.)

4. (2+3+2+3 pont)
Legyen $f(x) = sgn(x) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, ha $x \in (-\pi, \pi)$ Mennyi $\hat{f}_{-5}$ es $\hat{f}_4$?
$y'_t(t,x) = y''_{xx}(t,x),\ y(0,x) = 8e^{9ix} + \sin x$. Mennyi $y(t,x)$?
$y'_t(t,x) = y''_{xx}(t,x),\ y(0,x) = \sum_{n\in\mathbb{Z}} \frac{1}{n} \frac{e^{inx}}{\sqrt{2\pi}}$. Ird fel $y(t,x)$-et vegtelen sor alakban!
$y''_{tt}(t,x) = y''_{xx}(t,x),\ y(0,x) = 8e^{9ix},\ y'(0,x) = \cos(5x) + e^{9ix}$. Mennyi $y(t,x)$?

2. (1+1+1+5+2 pont)
Keresd meg a kovetkezo DE altalanos megoldasat! $y' = \frac{1}{3}\delta(x) + 1$.
Keresd meg a kovetkezo DE megoldasat az ($y(x) = 0$, ha $x < 0$) feltetel mellett! $2y'' = \delta(x)$.
Keresd meg a kovetkezo DE megoldasat a ($G(x) = 0$, ha $x < 0$) feltetel mellett!
$2G'' + 10G' + 12G = \delta(x)$.
Add meg a kovetkezo mennyisegeket!
$G(0^+) - G(0^-) =$ $G'(0^+) - G'(0^-) =$
Mennyi $G(x)$?
Ird fel az $y'' + 4y' + 13y = f(x)$ DE megoldasat, ha ($y(x) = f(x) = 0$, ha $x \ll 0$).

3. (2+2+3+2+1 pont)
Szamold ki az $f = e^{6t}H(t-5)$ fuggveny Laplace transzformaltjat a definicio alapjan!
Szamold ki az alabbi fuggvenypar $f{*}g$ konvoluciojat! $f(t) = e^{6t},\ g(t) = e^{7t}$
Mennyi $\mathcal{L}(f{*}g) - \mathcal{L}(f)\mathcal{L}(g)$?
Legyen $2y'' + 10y' + 12y = e^{-t} + e^{-3t}$, $y(0) = 3,\ y'(0) = 4$. Mennyi $\mathcal{L}(y(t)) = Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$.)
Ird fel azt, hogy hogyan nez ki $Y(s)$ parcialis tort felbontasa!
Mennyi $y(t)$? ($\mathcal{L}(e^{at}) = \frac{1}{s-a}$.)

2. (1+1+1+5+2 pont)

Keresd meg a kovetkezo DE altalanos megoldasat! $y' = \frac{1}{3}\delta(x) + 1$.

$$y(x) = \frac{1}{3} H(x) + x + C$$

Keresd meg a kovetkezo DE megoldasat az ($y(x) = 0$, ha $x < 0$) feltetel mellett! $2y'' = \delta(x)$.

$$y'' = \frac{1}{2}\delta$$
$$y' = \frac{1}{2} H \qquad y(x) = \begin{cases} 0, & \text{ha } x < 0 \\ \frac{1}{2}x, & \text{ha } x > 0 \end{cases}$$

Megjegyzés: $y(0^+) - y(0^-) = 0$

$y'(0^+) - y'(0^-) = \frac{1}{2}$

Ezeket az adatokat kell majd ITT használni!

Keresd meg a kovetkezo DE megoldasat a ($G(x) = 0$, ha $x < 0$) feltetel mellett!

$2G'' + 10G' + 12G = \delta(x)$.

Add meg a kovetkezo mennyisegeket!

$G(0^+) - G(0^-) =$ 0 $\qquad G'(0^+) - G'(0^-) =$ $\frac{1}{2}$

Mennyi $G(x)$? Ha $x < 0$, $G(x) = 0$

Ha $x > 0$, $G(x) = y(x)$, ahol $y(x)$ megoldása az

$2y'' + 10y' + 12y = 0$, $y(0) = 0$, $y'(0) = \frac{1}{2}$ DE-nek.

$$2\lambda^2 + 10\lambda + 12 = 0 \longrightarrow \lambda_1 = -2, \ \lambda_2 = -3$$
$$y = C_1 e^{-2x} + C_2 e^{-3x}, \quad C_1 + C_2 = 0, \ -2C_1 - 3C_2 = \frac{1}{2}$$
$$C_1 = \frac{1}{2}, \ C_2 = -\frac{1}{2}$$

$$G(x) = \begin{cases} 0, & \text{ha } x < 0 \\ \frac{1}{2}e^{-2x} - \frac{1}{2}e^{-3x}, & \text{ha } x > 0 \end{cases}$$

$2y'' + 10y' + 12y = f(x)$ *

Ird fel az ~~$y'' + 4y' + 13y$~~ $= f(x)$ DE megoldasat, ha ($y(x) = f(x) = 0$, ha $x \ll 0$).

$$y(x) = \int_{-\infty}^{\infty} G(x-z) f(z)\, dz =$$
$$= \int_{-\infty}^{x} \left(\frac{1}{2} e^{-2(x-z)} - \frac{1}{2} e^{-3(x+z)} \right) f(z)\, dz$$

*: Ez bizonyos értelemben sajtóhiba, itt elfogadható megoldásnak számít az előző pontban megkapott G használata, vagy az eredeti példa értelmes megoldási kísérlete.

1+1

3. (2+[redacted]+3+2+1 pont)

Szamold ki az $f = e^{6t}H(t-5)$ fuggveny Laplace transzformaltjat a definicio alapjan!

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} \cdot e^{6t} \cdot H(t-5)\,dt = \int_5^\infty e^{(-s+6)t}\,dt = \left[\frac{e^{(-s+6)t}}{-s+6}\right]_5^\infty$$

$$= 0 - \frac{e^{(-s+6)\cdot 5}}{-s+6} = \frac{e^{5(6-s)}}{s-6} \quad (\text{ha } \operatorname{Re} s > 6)$$

Szamold ki az alabbi fuggvenypar $f*g$ konvoluciojat! $f(t) = e^{6t}$, $g(t) = e^{7t}$

$$(f*g)(t) = \int_0^t e^{6(t-\tau)} \cdot e^{7\tau}\,d\tau = \int_0^t e^{6t+\tau}\,d\tau = \left[e^{6t+\tau}\right]_0^t =$$

$$= e^{7t} - e^{6t}$$

$$\mathcal{L}(e^{7t} - e^{6t}) = \frac{1}{s-7} - \frac{1}{s-6}$$

$$\mathcal{L}(e^{7t}) \cdot \mathcal{L}(e^{6t}) = \frac{1}{s-7} \cdot \frac{1}{s-6}$$

egyenlőek!

Ez automatikusan nulla!

Mennyi $\mathcal{L}(f*g) - \mathcal{L}(f)\mathcal{L}(g)$? =

Legyen $2y'' + 10y' + 12y = e^{-t} + e^{-3t}$, $y(0) = 3$, $y'(0) = 4$. Mennyi $\mathcal{L}(y(t)) = Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$.)

$$2(s^2 Y(s) - 3s - 4) + 10(sY(s) - 3) + 12Y(s) = \frac{1}{s+1} + \frac{1}{s+3}$$

$$Y(s) = \frac{1}{2s^2 + 10s + 12} \cdot \left(6s + 38 + \frac{1}{s+1} + \frac{1}{s+3}\right)$$

$$2s^2 + 10s + 12 = 0$$

$$s_1 = -2, \quad s_2 = -3$$

Ird fel azt, hogy hogyan nez ki $Y(s)$ parcialis tort felbontasa!

$$Y(s) = \frac{\text{polinom}(s)}{(s+2)(s+3)^2(s+1)} = \frac{A}{s+2} + \frac{B}{(s+3)^2} + \frac{C}{s+3} + \frac{D}{s+1}$$

Mennyi $y(t)$? ($\mathcal{L}(e^{at}) = \frac{1}{s-a}$.)

$$y(t) = \mathcal{L}^{-1}(Y(s)) = Ae^{-2t} + B \cdot t \cdot e^{-3t} + C \cdot e^{-3t} + De^{-1 \cdot t}$$

$$\left(\mathcal{L}(t) = \int_0^\infty e^{-st} t\,dt = \frac{1}{s^2}, \quad \mathcal{L}(e^{-3t} \cdot t) = \int_0^\infty e^{-st} \cdot e^{-3t} \cdot t\,dt = \int_0^\infty e^{-(s+3)t} \cdot t\,dt = \frac{1}{(s+3)^2} \right)$$

*A megoldás ezen tag kiszámolása nélkül is helyes

4. (2+3+2+3 pont)

Legyen $f(x) = sgn(x) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, ha $x \in (-\pi, \pi)$ Mennyi $\hat{f}_{-5}$ es ~~X~~?

$$\hat{f}_{-5} = \left(\frac{e^{i(-5)x}}{\sqrt{2\pi}}, sgn(x)\right) = \int_{-\pi}^{\pi} \overline{\frac{e^{-5ix}}{\sqrt{2\pi}}} \cdot sgn x \, dx = \int_{-\pi}^{0} \frac{e^{5ix}}{\sqrt{2\pi}} \cdot (-1)\, dx +$$

$$+ \int_{0}^{\pi} \frac{e^{5ix}}{\sqrt{2\pi}} \cdot 1\, dx = -\frac{1}{\sqrt{2\pi}} \left[\frac{e^{5ix}}{5i}\right]_{-\pi}^{0} + \frac{1}{\sqrt{2\pi}} \left[\frac{e^{5ix}}{5i}\right]_{0}^{\pi} =$$

$$-\frac{1}{\sqrt{2\pi}\cdot 5i}\Big(1 - \underbrace{e^{-5\pi i}}_{-1}\Big) + \frac{1}{\sqrt{2\pi}\cdot 5i}\Big(\underbrace{e^{+5\pi i}}_{-1} - 1\Big) = 4i \frac{1}{\sqrt{2\pi}\cdot 5} = \frac{2\sqrt{2}}{5} i \frac{1}{\sqrt{\pi}}$$

$y'_t(t,x) = y''_{xx}(t,x),\ y(0,x) = 8e^{9ix} + \sin x.$ Mennyi $y(t,x)$?

Mivel $\frac{\partial^2}{\partial x^2} e^{9ix} = -81 \cdot e^{9ix}$ és $\frac{\partial^2}{\partial x^2} \sin x = -1 \cdot \sin x$, így

$$y(t,x) = 8 \cdot e^{9ix} \cdot e^{-81\cdot t} + \sin x \cdot e^{-1\cdot t}$$

$y'_t(t,x) = y''_{xx}(t,x),\ y(0,x) = \sum_{n\in\mathbb{Z}} \frac{1}{n}\frac{e^{inx}}{\sqrt{2\pi}}$. Ird fel $y(t,x)$-et vegtelen sor alakban!

$$y(t,x) = \sum_{n\in\mathbb{Z}} \frac{1}{n} \cdot \frac{e^{inx}}{\sqrt{2\pi}} \cdot e^{-n^2 t}$$

$y''_{tt}(t,x) = y''_{xx}(t,x),\ y(0,x) = 8e^{9ix},\ y'(0,x) = \cos(5x) + e^{9ix}.$ Mennyi $y(t,x)$?

$$y(t,x) = 8 \cdot e^{9ix} \cdot \cos(9t) + \cos(5x) \cdot \frac{\sin(5t)}{5} + e^{9ix} \cdot \frac{\sin(9t)}{9}$$

[Pl.: Ha a $y''_{tt} = y''_{xx}$, $y(0,x) = 0$, $y'_t(0,x) = \cos(5x)$ DE megoldását $y(t,x) = \cos(5x) \cdot c(t)$ alakban keressük, akkor $y''_{xx} = -5^2 \cos(5x) \cdot c(t) = y''_{tt} = \cos(5x) \cdot c''(t)$, vagyis

$-5^2 c(t) = c''(t) \longrightarrow c(t) = C_1 \cos(5t) + C_2 \sin(5t)$.

Továbbá $y(0,x) = 0 \longrightarrow C_1 = 0$

$y'_t(0,x) = \cos(5x) \longrightarrow C_2 = \frac{1}{5}$]

1. $x_0 = 9$, $x_{n+1} = \phi(x_n) = 5x_n + 8$, Mennyi x_n?
Megoldas: ϕ fixpontja: $x_f = 5x_f + 8, \implies x_f = -2$. Fixpont koruli linearizalt dinamika:

$$\Delta x_n = x_n - x_f = x - (-2), \quad \Delta x_{n+1} = 5\Delta x_n.$$

Tehat

$$x_n = 5^n \left(9 - (-2)\right) + (-2).$$

2. Legyen $f(x) = x$, ha $x \in [0,2]$, maskulonben $f = 0$. Ha $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(p) e^{ipx}\, dp$, akkor mennyi $\hat{f}(3)$?
Megoldas:

$$\hat{f}(3) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\cdot 3\cdot x}\, dx = \frac{1}{\sqrt{2\pi}} \int_0^2 x e^{-i\cdot 3\cdot x}\, dx$$

$$= \frac{e^{-3ix}\left(\frac{1}{9} + \frac{ix}{3}\right)}{\sqrt{2\pi}}\bigg|_0^2 = \frac{-1 + (1+6i)e^{-6i}}{9\sqrt{2\pi}}$$