1. (3+2+3+2)
The set of functions $\{\sin(nx)\,|\,n=1,2,\dots\}$ forms an othogonal basis in $\mathcal{H}=L^2([0,\pi])$. Give an orthonormed basis in $\mathcal{H}$!

Let $f^2([-\pi,\pi])$ Let $f(x)=3$, if $x\in[-1,0]$, otherwise assume that $f(x)=0$, if $x\in[-\pi,\pi]\setminus[-1,0]$. If $f(x)=\frac{1}{\sqrt{2\pi}}\sum_{n\in\mathbb{Z}}\hat{f}_n e^{inx}$, then how much is $\hat{f}(-3)$?

$\phi_t(t,x)=\phi_{xx}(t,x),\ \phi(0,x)=\exp(i(5x))+\sin(2x).$ How much is $\phi(t,x)$?

$\phi_t(t,x)=\phi_{xx}(t,x),\ \phi(0,x)=\sin(3x)+5.$ How much is $\phi(t,x)$?

2. (2+1+(2+4+1) pont)
If $y''+7y'+8y=e^{i\omega t}$, $y=A(\omega)e^{i\omega t}$, then how much is $A(\omega)$?

Let $f(x)=2$, if $x\in[-1,1]$, otherwise $f=0$. If $f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\hat{f}(p)e^{ipx}\,dp$, then how much is $\hat{f}(-0.5)$?

Solve the following DE with the condition ($G(t)=0$, if $t<0$) !
$9G''+27G=\delta.$
Compute the following quantities!
$G(0^+)-G(0^-)=$ $\qquad$ $G'(0^+)-G'(0^-)=$

How much is $G(t)$?

What is the solution of $9y''+27y=f(t)$, if ($y(t)=f(t)=0$, ha $t\ll 0$) ?

3. (2+2+(3+2+1) pont)
Compute the Laplace transform of $f=1-H(t-1)$!

Compute the convolution $f*g$! $\quad f(t)=2,\ \ g(t)=\sin 5t.$

$y''-25y=(1-t)^2,\quad y(0)=2,\ y'(0)=3.$ Compute $\mathcal{L}(y(t))=Y(s)$! ($\mathcal{L}(t^n)=\frac{n!}{s^{n+1}}$.)

Write down the partial fraction decomposition of $Y(s)$! (Do not compute the coefficients!)

How much is $y(t)$?

4.(1+1+2+3+3 pont)

$$A=\begin{pmatrix}2i & -3i\\ 3 & 4i\end{pmatrix}.$$

Compute A^* ?

Let $f_1=(i/\sqrt{2},i/\sqrt{2})^T$, $f_2=(z,-1/\sqrt{2})^T$ be an othonormed basis. How much is z ?

$v=(2,3)^T$ can be expressed as $\alpha f_1+\beta f_2$! How much is α ?

Assume that $\phi(t,x)$ satisfies $\partial^2_{tt}\phi-\partial^2_{tx}\phi+8\partial^2_{xx}\phi=0$. If $\phi(t,x)=e^{i(\omega t+kx)}$, find a relation between ω and k !

Let

$$\frac{d}{dt}\bar{y}+\begin{pmatrix}2 & 3\\ 3 & 2\end{pmatrix}\bar{y}=\begin{pmatrix}t^2\\ t^4\end{pmatrix},\quad \bar{y}(0)=\begin{pmatrix}0\\ 0\end{pmatrix}.$$

How much is $\bar{Y}(s)$? (The computation of the inverse matrix is not required.)