

3a. (1+1+1+2 pont)

$$y' = (-y^2 + 4).$$

Find the fixed points of the DE!

If $y(0) = 0$, how much is

$$\lim_{x \rightarrow \infty} y(x) = \quad \lim_{x \rightarrow -\infty} y(x) = ?$$

Sketch the $y(x)$ solution curves of the DE!

3b. (2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -y_1 + 2 \\ 2y_2(-y_1 - 3y_2) \end{pmatrix}$$

Find the fixed points of the DE!

Write down the linearized DEs around the fixed points!

1, Diff.Eq. 2016.03.22. NEPTUN:

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1. (4+(3+3) pont)

a)

$$y' = f(x, y) = x^2 + y - 2;$$

How much is y'' ? Write down the second order Taylor polynomial of $y(x)$ around $x = 0$, if $y(0) = 3$!

b) Apply the Euler and the Heun methods on the following DEs with stepsize $\Delta x = 0.01$!

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} y_1 - y_2 \\ 3y_2^2 + x \end{pmatrix}, \quad \bar{y}(2) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

What are the predictions for $\bar{y}(2.01)$?

Euler:

Heun:

2. (5+2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -y_1 - 2y_2 \\ 2y_1 - y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Find the eigenvalues and eigenvectors of A !

Write down the general solution of the DE!

Compute the particular solution!

(5 × 2 pont)

$$A = \begin{pmatrix} -1 & 0 \\ 6 & -1 \end{pmatrix}$$

How much is e^{xA} ?

Express the solution of the following DE

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -y_1 \\ 6y_1 - y_2 \end{pmatrix}, \quad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

with e^{xA} !

2b) Express the following DE as a first order system!

$$\frac{d^2}{dx^2} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_1'^2 - y_2 \\ 2y_2' - 3y_1' \end{pmatrix}$$

How much is $z = e^{-1-i\pi}$?