1a. (1+1+1+2 pont) $f'(1)$

$y' = (-y^3 + y). = f(y) = -y(y^2+1)$ $y-1=\Delta y_3$

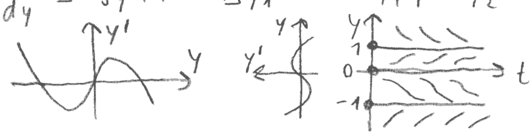
1 Keresd meg a DE fixpontjait! $f(y)=0 \longrightarrow y_1=-1,\ y_2=0,\ y_3=1$

1 Irđ fel a fixpontok koruli linearizalt kozelito DE-t! $\frac{df(y)}{dy} = -3y^2+1$ $\Delta y_1' = -2\Delta y_1,\ \Delta y_2' = 1\cdot\Delta y_2,\ \Delta y_3' = -2\Delta y_3$

1 Ha $y(0) = 0.1$, mennyi

$\lim_{x\to\infty} y(x) = 1$ $\lim_{x\to-\infty} y(x) = 0$

2 Vazold a DE megoldasgorbeit!



1b. (2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2y_2(-y_1 - 3y_2) \\ -y_1 + 2. \end{pmatrix}$$ Fixpont: $\begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -2/3 \end{pmatrix}$

2 Keresd meg a DE fixpontjait! $Jac = \begin{pmatrix} -2y_2 & -2y_1-12y_2 \\ -1 & 0 \end{pmatrix}$, $Jac(P_1) = \begin{pmatrix} 0 & -4 \\ -1 & 0 \end{pmatrix}$

3 Ird fel a fixpont koruli linearizalt kozelito DE-t! $Jac(P_2) = \begin{pmatrix} 4/3 & 4 \\ -1 & 0 \end{pmatrix}$

$$\frac{d}{dt}\begin{pmatrix} y_1-2 \\ y_1-0 \end{pmatrix} = \frac{d}{dt}\overline{\Delta y_1} = \begin{pmatrix} 0 & -4 \\ -1 & 0 \end{pmatrix}\overline{\Delta y_1},\quad \frac{d}{dt}\overline{\Delta y_2} = \begin{pmatrix} 4/3 & 4 \\ -1 & 0 \end{pmatrix}\overline{\Delta y_2}$$

$y \approx 3 + 7x + \frac{42}{2}x^2$

2. (4+(3+3) pont)

$y(0)=3$

a)

$y'(0) = 0^3 + 3^2 - 2 = 7$

1 $y'' = (\frac{\partial}{\partial x} + f\frac{\partial}{\partial y})f = 3x^2 + (x^3+y^2-2)\cdot 2y$

$y' = f(x, y) = x^3 + y^2 - 2;$ $y''(0) = 3\cdot 0^2 + (0^3+3^2-2)\cdot 2\cdot 3 = 42$ 3

Mennyi y'' ? Irđ fel y masodrendu Talyor polinomjat az $x = 0$ pont korul, ha $y(0) = 3$!

b) Alkalmazd az Euler, illetve a Heun modszert a kovetkezo DE-re $\Delta x = 0.01$ lepeskozzel! az $\bar{y}(2) =$ kezdeti feltetel mellett!

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} y_1 + 2y_2 \\ 3y_2^3 + x. \end{pmatrix}, \qquad \bar{y}(2) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Mit josol a ket modszer $\bar{y}(2.01)$-re? $\bar{y}(2{,}01) \approx \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2+2\cdot 3 \\ 3\cdot 3^3 + 2 \end{pmatrix}\cdot 0{,}01 = \begin{pmatrix} 2.08 \\ 3{,}83 \end{pmatrix}$ 3

Euler:

Heun: $\bar{y}(2{,}01) \approx \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 0{,}01\cdot\frac{1}{2}\left(\begin{pmatrix} 8 \\ 83 \end{pmatrix} + \begin{pmatrix} 2{,}08 + 2\cdot 3{,}83 \\ 3\cdot 3{,}83^3 + 2{,}01 \end{pmatrix}\right)$ 3

3. (5+2+3 pont)

$A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ $\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} y_1 - 2y_2 \\ 2y_1 + y_2 \end{pmatrix} = A\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Keresd meg A sajatertekeit es sajatvektorait!

5 $\begin{vmatrix} 1-\lambda & -2 \\ 2 & 1-\lambda \end{vmatrix} = 0 \longrightarrow \lambda_1 = 1+2i,\ \lambda_2 = 1-2i,$ $\bar{v}_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix}, \bar{v}_2 = \begin{pmatrix} 1 \\ i \end{pmatrix}$

2 Irđ fel a DE altalanos megoldasat! $\bar{y} = C_1 e^{(1+2i)x}\begin{pmatrix} 1 \\ -i \end{pmatrix} + C_2 e^{(1-2i)x}\begin{pmatrix} 1 \\ i \end{pmatrix}$

3 Szamold ki a DE partikularis megoldasat! $\frac{1+3i}{2} e^{(1+2i)x}\begin{pmatrix} 1 \\ -i \end{pmatrix} + \left(\frac{1-3i}{2}\right) e^{(1-2i)x}\begin{pmatrix} 1 \\ i \end{pmatrix}$

$C_1\begin{pmatrix} 1 \\ -i \end{pmatrix} + C_2\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \longrightarrow$

4. (~~5 x 2 pont~~nt)

$e^{xA} = e^{\begin{pmatrix} 2x & 0 \\ 6x & 2x \end{pmatrix}} = e^{\begin{pmatrix} 2x & 0 \\ 0 & 2x \end{pmatrix}} e^{\begin{pmatrix} 0 & 0 \\ 6x & 0 \end{pmatrix}} =$ $A = \begin{pmatrix} 2 & 0 \\ 6 & 2 \end{pmatrix}$ $= \begin{pmatrix} e^{2x} & 0 \\ 0 & e^{2x} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 6x & 1 \end{pmatrix} = \begin{pmatrix} e^{2x} & 0 \\ e^{2x}\cdot 6x & e^{2x} \end{pmatrix}$ 4

4 Mennyi e^{xA}?

2 Irđ fel a

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2y_1 \\ 6y_1 + 2y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$ $\bar{y} = \begin{pmatrix} e^{2x} & 0 \\ e^{2x}6x & e^{2x} \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ 2

DE partikularis megoldasat e^{xA} segitsegevel!

2 2b) Irđ at a kovetkezo DE rendszert elsorendu DE rendszerre! $\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \\ v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_1^2 - y_2 \\ 2v_2 - 3v_1 \end{pmatrix}$ 2

$$\frac{d^2}{dx^2}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_1'^2 - y_2 \\ 2y_2' - 3y_1' \end{pmatrix}$$

2 Mennyi $z = e^{1-i\pi}$? $= e\cdot(\cos(-\pi) + i\sin(-\pi)) = -e$ 2