16.majus.23. Diff.egy.v2.

1. (2+2+4+2) $\mathcal{L}(f*g)=\mathcal{L}(f)\cdot\mathcal{L}(g)=\frac{3!}{s^4}\cdot\frac{4!}{s^5}=\frac{144}{s^9}$

2 • Legyen $f(t)=t^3$, $g(t)=t^4$, $\mathcal{L}(t^n)=n!/s^{n+1}$. Mennyi $f*g$ Laplace transzformaltja?

2 • Legyen $f_1=(i/2,\sqrt{3}/2)^T$, $f_2=(\sqrt{3}/2,i/2)^T$ egy ortonormalt bazis. A $v=(1,2)^T$ vektor kifejezheto az f-ek linearis $\alpha f_1+\beta f_2$ kombinaciojakent! Mennyi α ? $\alpha=(f_1,v)=\overline{i/2}\cdot 1+\sqrt{3}/2\cdot 2=\sqrt{3}-\frac{1}{2}i$

4 • $y''=\left(\frac{\partial}{\partial x}+f(x,y)\frac{\partial}{\partial y}\right)f(x,y)$ $(\partial_x+(x+y^2+1)\partial_y)(x+y^2+1)=1+(x+y^2+1)\cdot 2y$ $y(1)=3,\ y'(1)=1+3^2+1=11$

$y'=f(x,y)=x+y^2+1;$ $y''(1)=1+(1+3^2+1)\cdot 2\cdot 3=67$

Mennyi y'' ? Ird fel y masodrendu Talor polinomjat az $x=1$ pont korul, ha $y(1)=3$! $y(1+\Delta x)\approx 3+11\Delta x+\frac{67}{2}\Delta x^2$

2 • Legyen $f(t)=t-1, g(t)=t$. Mennyi $(f*g)(t)$? $=\int_0^t[(t-\tau)-1]\cdot\tau\, d\tau=\int_0^t(t-1)\tau-\tau^2\, d\tau=(t-1)\cdot\frac{t^2}{2}-\frac{t^3}{3}=$ $=t^3/6-t^2/2$

2. (2+2)+(1+4+1) pont $s^2Y(s)-2s-3+16\cdot Y(s)=\frac{2!}{s^3}-6\cdot\frac{1}{s^2}+9\cdot\frac{1}{s}$ $Y(s)=\frac{1}{s^2+16}\left(2s+3+\frac{2}{s^3}-\frac{6}{s^2}+\frac{9}{s}\right)$

2 • Legyen $y''+16y=(t-3)^2$, $y(0)=2$, $y'(0)=3$. Mennyi $\mathcal{L}(y(t))=Y(s)$? ($\mathcal{L}(t^n)=\frac{n!}{s^{n+1}}$.)

2 Ird fel azt, hogy hogyan nez ki $Y(s)$ parcialis tort felbontasa! (Az egyutthatokat nem kell kiszamolni!) $Y(s)=\frac{A}{s-4i}+\frac{B}{s+4i}+\frac{C}{s^3}+\frac{D}{s^2}+\frac{E}{s}$

• Oldd meg a kovetkezo DE-t a kovetkezo kezdeti feltetelek mellett: ($G(t)=0$, if $t<0$) ! $G''+9G=2\delta$.

1 Mennyi $G(0^+)-G(0^-)=0$ $G'(0^+)-G'(0^-)=2$ $G(t)=\begin{cases}0, & \text{ha } t<0\\ 2\cdot\frac{1}{3}\sin(3t), & \text{ha } t>0\end{cases}$

4 Mennyi $G(t)$? $y(t)=\int_{-\infty}^{\infty}G(t-\tau)f(\tau)\,d\tau=\int_{-\infty}^{t}\frac{2}{3}\sin(3(t-\tau))f(\tau)\,d\tau$

1 Mennyi a megoldasa a $y''+9y=2f(t)$ DE-nek, ha ($y(t)=f(t)=0$, ha $t\ll 0$) ?

3. (1+1+1+2 pont) + (2+3 pont)

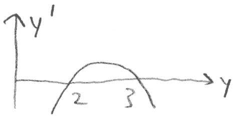
• $y'=(2-y)(y-3)$. $y_1=2, y_2=3$ $\frac{d}{dy}[(2-y)(y-3)]=-2y+5$

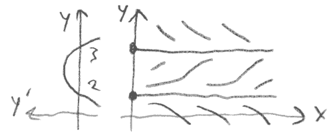
1 Keresd meg a DE fixpontjait! $\frac{d}{dx}(y-2)=\frac{d}{dx}\Delta y_1=[-2\cdot(2)+5]\Delta y_1=1\cdot\Delta y_1$

1 Ird fel a linearizalt kozelito DE-ket a fixpontok korul! $\frac{d}{dx}(y-3)=\frac{d}{dx}\Delta y_2=-1\cdot\Delta y_2$

1 Ha $y(0)=2.3$, akkor mennyi

$\lim_{x\to\infty}y(x)=3$ $\lim_{x\to-\infty}y(x)=$? 2





2 Rajzold le az $y(x)$ megoldasgorbeit a DE-nek!

• Fixpont: $y_1+3=0\to y_1=-3$, $2\cdot y_2\cdot(-3)=0\to y_2=0$ $P=\begin{pmatrix}-3\\0\end{pmatrix}$ $\begin{pmatrix}y_1'\\y_2'\end{pmatrix}=\begin{pmatrix}y_1+3\\2y_2y_1.\end{pmatrix}$ $Jac=\begin{pmatrix}\partial_{y_1}(y_1+3) & \partial_{y_2}(y_1+3)\\ \partial_{y_1}(2y_2y_1) & \partial_{y_2}(2y_2y_1)\end{pmatrix}=\begin{pmatrix}1 & 0\\ 2y_2 & 2y_1\end{pmatrix}$

2 Keresd meg a DE fixpontjat!

3 Ird fel a fixpont koruli linearizalt kozelito DE-t! $Jac(P)=\begin{pmatrix}1&0\\0&-6\end{pmatrix}$ $\frac{d}{dx}\begin{pmatrix}y_1-(-3)\\y_2-0\end{pmatrix}=\frac{d}{dx}\overline{\Delta y}=\begin{pmatrix}1&0\\0&-6\end{pmatrix}\overline{\Delta y}$

4. (3+3)+(3+1) pont

• sajátérték: $\begin{vmatrix}-1-\lambda & 2\\ 2 & -1-\lambda\end{vmatrix}=0\to\lambda_1=1,\ \lambda_2=-3.$ sajátvekt: $\begin{pmatrix}-1-1 & 2\\ 2 & -1-1\end{pmatrix}\begin{pmatrix}u\\v\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\to u=v,\ \overline{v}_1=\begin{pmatrix}1\\1\end{pmatrix}$

$\overline{v}_1=\begin{pmatrix}1\\1\end{pmatrix},\ \overline{v}_2=\begin{pmatrix}1\\-1\end{pmatrix}$ $\begin{pmatrix}y_1'\\y_2'\end{pmatrix}=\begin{pmatrix}-y_1+2y_2\\2y_1-y_2\end{pmatrix}=A\begin{pmatrix}y_1\\y_2\end{pmatrix}$, $\begin{pmatrix}y_1(0)\\y_2(0)\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}$

3 Keresd meg A sajatertekeit es sajatvektorait! $\overline{y}_{ált}=c_1e^{1\cdot x}\begin{pmatrix}1\\1\end{pmatrix}+c_2e^{-3x}\begin{pmatrix}1\\-1\end{pmatrix}$

3 Ird fel a partikularis megoldast! $\begin{pmatrix}1\\3\end{pmatrix}=c_1\begin{pmatrix}1\\1\end{pmatrix}+c_2\begin{pmatrix}1\\-1\end{pmatrix}\longrightarrow\overline{y}_{part}=2\cdot e^{x}\begin{pmatrix}1\\1\end{pmatrix}-e^{-3x}\begin{pmatrix}1\\-1\end{pmatrix}$

• $e^{xB}=e^{\begin{pmatrix}-x&0\\0&-x\end{pmatrix}+\begin{pmatrix}0&x\\0&0\end{pmatrix}}=$

$=\begin{pmatrix}e^{-x}&0\\0&e^{-x}\end{pmatrix}\cdot\left[\begin{pmatrix}1&0\\0&1\end{pmatrix}+\begin{pmatrix}0&x\\0&0\end{pmatrix}+\frac{1}{2}\underbrace{\begin{pmatrix}0&x\\0&0\end{pmatrix}^2}_{=0}+\dots\right]=$ $B=\begin{pmatrix}-1&1\\0&-1\end{pmatrix}$ $\begin{pmatrix}e^{-x}&0\\0&e^{-x}\end{pmatrix}\begin{pmatrix}1&x\\0&1\end{pmatrix}=\begin{pmatrix}e^{-x}&xe^{-x}\\0&e^{-x}\end{pmatrix}$

3 Mennyi e^{xB}?

Ird fel a

$$\begin{pmatrix}y_1'\\y_2'\end{pmatrix}=\begin{pmatrix}-y_1+y_2\\-y_2\end{pmatrix},\quad\begin{pmatrix}y_1(0)\\y_2(0)\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}$$

1 DE partikularis megoldasat e^{xB} segitsegevel!

$\overline{y}_{part}=e^{xB}\begin{pmatrix}1\\3\end{pmatrix}=\begin{pmatrix}e^{-x}&xe^{-x}\\0&e^{-x}\end{pmatrix}\begin{pmatrix}1\\3\end{pmatrix}=e^{-x}\cdot\begin{pmatrix}1+3x\\3\end{pmatrix}$