1. (3+2+2+3) Pl.: $\frac{e^{i\frac{2\pi}{2}nx}}{\sqrt{2}}, n \in \mathbb{Z}$

3 Add meg egy ortonormalt bazisat $\mathcal{H} = L^2([0,2], dx)$-nek.

2 Legyen $f(t) = t^2$, $g(t) = t^3$, $\mathcal{L}(t^n) = n!/s^{n+1}$. Mennyi $f * g$ Laplace transzformaltja? $\mathcal{L}(f*g) = \mathcal{L}(f)\cdot\mathcal{L}(g) = \frac{2!}{s^3}\cdot\frac{3!}{s^4} = \frac{12}{s^7}$

2 $\phi_t(t,x) = \phi_{xx}(t,x)$, $\phi(0,x) = \cos(\cancel{x}(5x)) + \exp(2x\cancel{i}) + 4$. Mennyi $\phi(t,x)$? $= e^{-25t}\cos(5x) + e^{-4t}e^{2ix} + 4$

3 $\phi_{tt}(t,x) = \phi_{xx}(t,x)$, $\phi(0,x) = 1 + \sin(3x)$, $\phi_t(0,x) = -\sin(3x)$. Mennyi $\phi(t,x)$?

$$\varphi(t,x) = 1 + \cos(3t)\sin(3x) - \frac{\sin(3t)}{3}\sin(3x)$$

2. (2+1+(2+4+1) point)

2 Ha $y''' + 7y' - 8y = e^{i\omega t}$, $y = A(\omega)e^{i\omega t}$, akkor mennyi $A(\omega)$? $A(\omega) = \frac{1}{(i\omega)^3 + 7i\omega - 8} = \frac{1}{-i\omega^3 + 7i\omega - 8}$

1 Legyen $f(\cancel{t}) = H(t-3)H(3-t)$. Ha $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\hat{f}(p)e^{ipx}\,dp$,

akkor mennyi $\hat{f}(7)$? $H(t-3)$ $H(3-t)$ tehát $f(t) = 0 \longrightarrow \hat{f}(7) = 0$

Oldd meg a kovetkezo DE-t a kovetkezo kezdeti feltetelek mellett: ($G(t) = 0$, if $t < 0$) !

$G''/2 + 18G = \delta$. ha $t \neq 0 \longrightarrow G''(t) + 36G(t) = 0$

Szamitsd ki! $\longrightarrow G(t) = C_1\cos(6t) + C_2\sin(6t)$

2 $G(0^+) - G(0^-) = 0$ $\qquad G'(0^+) - G'(0^-) = 2$

4 Mennyi $G(t)$? Tehát $G(t) = \begin{cases} 0, & \text{ha } t < 0 \\ \frac{\sin(6t)}{6}, & \text{ha } t > 0 \end{cases}$

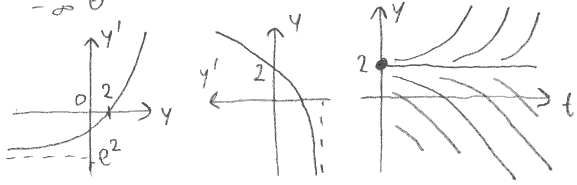
1 Mennyi a megoldasa a $0.5y'' + 18y = f(t)$ DE-nek, ha ($y(t) = f(t) = 0$, ha $t \ll 0$) ?

$$y(t) = \int_{-\infty}^{t} G(t-\tau)f(\tau)\,d\tau = \int_{-\infty}^{t}\frac{1}{6}\sin(6(t-\tau))f(\tau)\,d\tau$$

3a. (1+1+1+2 point)

$y' = (e^y - e^2)$.

1 Keresd meg a DE fixpontjait! $e^y - e^2 = 0 \longrightarrow y_1 = 2$



Ha $y(0) = 1$, akkor mennyi

1+1 $\lim_{x\to\infty} y(x) = -\infty$, $\lim_{x\to-\infty} y(x) = ?2$

2 Rajzold le az $y(x)$ megodasgorbeit a DE-nek!

$$Jac = \begin{pmatrix} \partial_{y_1}(e^{-y_1+3}-1) & \partial_{y_2}(e^{-y_1+3}-1) \\ \partial_{y_1}(2y_2y_1) & \partial_{y_1}(2y_2y_1) \end{pmatrix} = \begin{pmatrix} e^{-y_1+3} & 0 \\ 2y_2 & 2y_1 \end{pmatrix}$$

3b. (2+3 point) $P = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Fixpont: $e^{-y_1+3} - 1 = 0 \rightarrow y_1 = 3$

$2y_2\cdot y_1 = 2\cdot y_2\cdot 3 = 0 \rightarrow y_2 = 0$

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} e^{-y_1+3} - 1 \\ 2y_2y_1 \end{pmatrix} \qquad Jac(P) = \begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$$

2 Keresd meg a DE fixpontjait!

3 Irđ fel a fixpontok koruli linearizalt kozelito DE-t! $\frac{d}{dt}\begin{pmatrix} y_1 - 3 \\ y_2 - 0 \end{pmatrix} = \frac{d}{dt}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix}$

4. (3+3)+(2+2) point)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -y_1 + 2y_2 \\ -3y_2 \end{pmatrix} = A\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

3 Keresd meg A sajatertekeit es sajatvektorait! $A = \begin{pmatrix} -1 & 2 \\ 0 & -3 \end{pmatrix}$, $\det(A - \lambda E) = \begin{vmatrix} -1-\lambda & 2 \\ 0 & -3-\lambda \end{vmatrix} = 0$

3 Irđ fel a partikularis megoldast!

$\lambda_1 = -1$, $\qquad \begin{pmatrix} -1-(-1) & 2 \\ 0 & -3-(-1) \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \longrightarrow v = 0$

$\lambda_2 = -3$, $\qquad \begin{pmatrix} -1-(-3) & 2 \\ 0 & -3-(-3) \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \longrightarrow \begin{matrix} 2u + 2v = 0 \\ u = -v \end{matrix}$

$\overline{V}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\overline{V}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\qquad \overline{y}_{ált} = C_1 e^{-1\cdot t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{-3\cdot t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$C_1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \longrightarrow \begin{matrix} C_2 = -3 \\ C_1 = 4 \end{matrix}$

$\overline{y}_{part} = 4e^{-t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 3e^{-3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

2 Legyen $f(t) = t^2$, $g(t) = 3$. Mennyi $(f * g)(t)$?

2 Legyen $f(t) = 1 - 0.3H(5-t)$. Mennyi $(\mathcal{L}(f))(s)$?

$$(f*g)(t) = \int_0^t (t-\tau)^2\cdot 3\,d\tau = \int_0^t 3\cdot\tau^2\,d\tau = \left[3\cdot\frac{\tau^3}{3}\right]_0^t = t^3$$

$f*g = g*f$

$$\mathcal{L}(f)(s) = \int_0^\infty e^{-st}(1 - 0.3H(5-t))\,dt = \int_0^\infty e^{-st}dt - 0.3\int_0^5 e^{-st}dt = \frac{1}{s} - 0.3\left[\frac{e^{-st}}{-s}\right]_0^5 = \frac{1}{s} - 0.3\left(\frac{e^{-5s}}{-s} - \frac{1}{-s}\right)$$