Test 1. Diff.Eq. 2015.04.15.

1a. $(1+1+1+2)$

$y' = (1 - y)y(1 + y)$.

Find the fixed pints of the DE!

Write down the linearized DE around the fixed points!

If $y(0) = 0$, how much is

$\lim_{x \to \infty} y(x) =$ $\qquad\qquad\qquad$ $\lim_{x \to -\infty} y(x) =$

Sketch the solution curves of the DE!

1b. $(2+3)$

$$\frac{d}{dt} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} y_2^2 \\ y_1^2 - 9 \\ -y_3 - 1 \end{pmatrix}.$$

Find the fixed pints of the DE!

Write down the linearized DE around the fixed points!

2. $(3+4+1+2)$

a)

$$y' = f(x, y) = y - x^2$$

How much are y'' es y''' ? Compute the third order Taylor polynom of y around $x = 2$, if $y(2) = 1$!

b) Apply the Euler and the Heun methods with $\Delta x = 0.1$ timestep for the following DE.

$$\frac{d}{dt} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_2 y_1 \\ y_1^2 - y_2^2 \end{pmatrix}. \qquad \begin{pmatrix} y_1(1) \\ y_2(1) \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

What are their predictions for $\bar{y}(1.1)$?

Euler:

Heun:

c) Solve it: $y' = -3y$, $y(78) = 97$.

d) Solve $y'' = 1 + x$, $y(-1) = 0 = y(1)$!

3. $y'' + 2y' + 2y = 0$. Express the second order DE as a first order system! Find the eigenvalues and eigenvectors of the coefficient matrix A !

Write down the general solution! Find the particular solution if $y(0) = 1$, $y'(0) = 0$ and when $y(0) = 0$, $y'(0) = 1$! What is the particular solution if $y(0) = 77$, $y'(0) = 88$?

4. $(5+2+3)$

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -4y_1 + 3y_2 \\ -4y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

Find the eigenvalues and eigenvectors of A !

Compute $\exp(tA)$!

Express

$$\begin{pmatrix} y_1(1) \\ y_2(1) \end{pmatrix}$$

with the help of the matrix exponential!