Test 1. Diff.Eq. 2015.04.15.

1a. (1+1+1+2)

$y' = (1-y)y$.

Find the fixed pints of the DE!

Write down the linearized DE around the fixed points!

If $y(0) = 0$, how much is

$\lim_{x\to\infty} y(x) =$ $\lim_{x\to-\infty} y(x) =$

Sketch the solution curves of the DE!

1b. (2+3)

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} y_2^2 - 4 \\ y_1^2 - 9 \\ -y_3 - 1 \end{pmatrix}.$$

Find the fixed pints of the DE!

Write down the linearized DE around the fixed points!

2. (3+4+1+2)

a)

$$y' = f(x,y) = y^3 + x^2$$

How much are y'' es y''' ? Compute the third order Taylor polynom of y around $x = 2$, if $y(2) = 1$!

b) Apply the Euler and the Heun methods with $\Delta x = 0.1$ timestep for the following DE.

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_2 + y_1 \\ y_1^2 + y_2^2 \end{pmatrix}. \qquad \begin{pmatrix} y_1(1) \\ y_2(1) \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

What are their predictions for $\bar{y}(1.1)$?

Euler:

Heun:

c) Solve it: $y' = 3y,\ y(7) = 9$.

d) Express the solution of $y' = \sin(3t^2),\ y(77) = 66$ with the help of definite integrals.

3. (5+2+3)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -4y_1 + 3y_2 \\ -3y_1 - 4y_2 \end{pmatrix} = A\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(1) \\ y_2(1) \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

Find the eigenvalues and eigenvectors of A !

Write down the general solution!

Compute

$$\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} \quad !$$

3. $((2+3)+5)$

a) Compute the Euler-Lagrange equations For L and M !

$$L = (2y')^3 - y^6 y', \qquad M = y_1'(y_2' + y_1) + y_1' y_2 - y_1 y_2.$$

b) Let $S[u] = \int_0^4 (y'(x))^2 + xy(x)\,dx$ where u is defined on $[0,3]$ and vanishes at the endpoints. Let V be defined on $[0,4]$, assume that it vanishes at the endpoints and is continuous. Assume also that elements of V are piecewise affine on the $[0,1],\ [1,2],\ [2,4]$ intervals. Let ϕ_1 and ϕ_2 be a basis of V, such that $\phi_1(1) = \phi_2(2) = 1$ and $\phi_2(2) = \phi_1(1) = 0$. Let $u_h = c_1\phi_1 + c_2\phi_2$. Compute the $S[u_h] = s(c_1, c_2)$ two variable function! (For the computation of the $xy(x)$ term in the integral use some approximate method!)