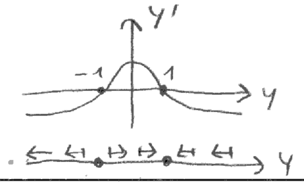


Diff.Egy.vizsga.V.25.

1a. (1+1+1+2 pont) $\frac{df(y)}{dy} = -\frac{2y}{(1+y^2)^2} = f'(y)$ $\quad f'(-1) = \frac{1}{2}$ $\quad f'(1) = -\frac{1}{2}$

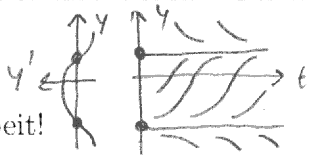
$y' = \frac{1}{1+y^2} - 1/2. = f(y)$

Keresd meg a DE fixpontjait! $y_1 = -1, \quad y_2 = 1$ $\quad \frac{d}{dt}(y-(-1)) = \frac{d}{dt}\Delta y = \frac{1}{2}\Delta y, \quad \frac{d}{dt}(y-1) = \frac{d}{dt}\Delta y = -\frac{1}{2}\Delta y$

Ird fel a fixpontok koruli linearizalt kozelito DE-t!

Ha $y(0) = 0$, mennyi

$\lim_{x\to\infty} y(x) = 1$ $\quad \lim_{x\to-\infty} y(x) = -1$



Vazold a DE megoldasgorbeit!

1b. (2+3 pont) Legyen

Euler: $y(2.1) \approx 3 + (3-2)\cdot 0.1 = 3.1$ $\quad$ Heun: $y(2.1) \approx 3 + \frac{1}{2}\left((3-2) + (3.1-2.1)\right)\cdot 0.1$

$$y' = f(x,y) = y - x.$$

Alkalmazd az Euler, illetve a Heun modszert $\Delta x = 0.1$ lepeskozzel, ha $y(2) = 3$. Mit josolnak ezek a modszerek $y(2.1)$-re?

---

2. (5+2+3 pont)

$\lambda_1 = 1, \; \bar{v}_1 = \begin{pmatrix}1\\0\end{pmatrix}$ $\quad \begin{pmatrix}y_1'\\y_2'\end{pmatrix} = \begin{pmatrix}y_1+3y_2\\4y_2\end{pmatrix} = \underbrace{A}_{\begin{pmatrix}1&3\\0&4\end{pmatrix}}\begin{pmatrix}y_1\\y_2\end{pmatrix}, \quad \begin{pmatrix}y_1(0)\\y_2(0)\end{pmatrix} = \begin{pmatrix}6\\8\end{pmatrix}$ $\quad \lambda_2 = 4, \; \bar{v}_2 = \begin{pmatrix}1\\1\end{pmatrix}$

Keresd meg A sajatertekeit es sajatvektorait! $\quad y_{ált} = C_1 e^{1\cdot t}\cdot\begin{pmatrix}1\\0\end{pmatrix} + C_2 e^{4t}\cdot\begin{pmatrix}1\\1\end{pmatrix}$

Ird fel a DE altalanos megoldasat! $\quad y_{part} = -2\cdot e^{1\cdot t}\cdot\begin{pmatrix}1\\0\end{pmatrix} + 8\cdot e^{4t}\cdot\begin{pmatrix}1\\1\end{pmatrix}$

Mennyi

$$\begin{pmatrix}y_1(1)\\y_2(1)\end{pmatrix} ? = \begin{pmatrix}-2e^1+8e^4\\8e^4\end{pmatrix}$$

3a. (1+2+2 pont) Legyen

$$A = \begin{pmatrix}2 & 2i\\-i & -3-i\end{pmatrix}. \quad A^* = \begin{pmatrix}2 & i\\-2i & -3+i\end{pmatrix}$$

Mennyi A^* ? $\quad (v,w) = \overline{(2+3i)}\cdot 4 + \overline{(4-i)}\cdot i = 7-8i$

Mennyi a $v = (2+3i, 4-i)^T$ es a $w = (4,i)^T$ vektorok belso szorzata? $\quad z = \frac{i}{2}$

Legyen $f_1 = (\sin(-30°), i\cos(-30°))^T$, $f_2 = (z, \cos(-30°))^T$ egy ortonormalt bazis. Mennyi z ?

A $v = (5,6)^T$ vektor kifejezheto az f-ek linearis $\alpha f_1 + \beta f_2$ kombinaciojakent! Mennyi α ? $= (f_1, v) = -3 - \frac{5\sqrt{3}}{2} i$

3b. (2+3 pont) $\frac{e^{3ix}+e^{-3ix}}{2}$ $\quad \hat{f}_{-2} = \sqrt{\frac{\pi}{2}} = \hat{f}_2$

Legyen $f(x) = \cos(3x) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, ha $x \in (-\pi,\pi)$. Szamold ki a nem nulla $\hat{f}_n$ egyutthatokat!

Szamitsd ki a Laplace tr. definicioja alapjan: $\mathcal{L}(\cos(2t)) = \frac{s}{s^2+2^2}$

4a. (3+1 pont) $\quad G(t) = \begin{cases} 0, & t<0\\ e^{-5t}, & t>0\end{cases}$

Oldd meg! $G' + 5G = \delta$, es $G(t) = 0$ negativ t-kre. $\quad y(t) = \int_0^t e^{-5(t-\tau)} f(\tau)\,d\tau$

Ird fel G segitsegevel az $y'' + 5y = f(t)$ egyenlet megoldasat, ha $y(t) = f(t) = 0$ negativ t-kre!

4b. (3+2+1 pont) $\quad Y(s) = \frac{1}{s+4}\left(7 + \frac{3}{s}\right) = \frac{A}{s+4} + \frac{B}{s} = \frac{6\frac{1}{4}}{s+4} + \frac{3/4}{s}$

$y' + 4y = 3, \; y(0) = 7.$

Szamold ki $y(t)$-nek az $Y(s)$ Laplace transzformaltjat!

Szamold ki $Y(s)$ parcialis tort felbontasat (az egyutthatokkal egyutt)! $\quad y(t) = 6\frac{1}{4}\cdot e^{-4t} + \frac{3}{4}$

Mennyi $\mathcal{L}^{-1}(Y(s))$?

Mennyi $y(t)$?