Vizsga, Diff.Egy., 2015.05.21. NEPTUN: Gyak.Vez.:

Név: Aláírás:

$5t^2-20t+20$

1. (3+2+1+3+1 pont)
$y''-4y=5(t-2)^2$, $y(0)=2$, $y'(0)=3$. Mennyi $Y(s)$? ($\mathcal{L}(t^n)=\frac{n!}{s^{n+1}}$)

$$\left(s^2Y(s)-2s-3\right)-4Y(s)=5\cdot\frac{2}{s^3}-20\cdot\frac{1}{s^2}+20\cdot\frac{1}{s}$$

$$Y(s)=\frac{1}{s^2-4}\left[2s+3+\frac{10}{s^3}-\frac{20}{s^2}+\frac{20}{s}\right]$$

Milyen $Y(s)$ parcialis tort felbontasanak a strukturaja?

$$Y(s)=\frac{A}{s-2}+\frac{B}{s+2}+\frac{C}{s^3}+\frac{D}{s^2}+\frac{E}{s}$$

Mennyi $y(t)$?

$$y(t)=Ae^{2t}+Be^{-2t}+\frac{C}{2}t^2+Dt+E$$

Oldd meg a $G''-4G=\delta(t)$ egyenletet, ahol $G(t)=0$, ha $t<0$!

$t\approx 0:\quad G''(t)\approx\delta(t)\longrightarrow G(0^+)-G(0^-)=0,\ G'(0^+)-G'(0^-)=1$

$t>0:\quad G''-4G=0\longrightarrow G(t)=Ae^{2t}+Be^{-2t}\overset{\checkmark}{=}\frac{1}{4}\left(e^{2t}-e^{-2t}\right)$

$$G(t)=\begin{cases}0, & \text{ha } t<0\\ \frac{1}{4}\left(e^{2t}-e^{-2t}\right), & \text{ha } t>0\end{cases}$$

Ird fel $G(t)$ segitsegevel az $y''-4y=f(t)$ egyenlet megoldasat, ha $y(t)=f(t)=0$ amikor $t<0$!

$$y(t)=\int_{-\infty}^{\infty}G(t-\tau)f(\tau)\,d\tau=\int_{-\infty}^{t}\frac{1}{4}\left(e^{2(t-\tau)}-e^{-2(t-\tau)}\right)f(\tau)\,d\tau$$

$$=\int_{0}^{t}\frac{1}{4}\left(e^{2(t-\tau)}-e^{-2(t-\tau)}\right)f(\tau)\,d\tau$$

4. (2+2+2+2+2 pont)

Legyen $y'' + 3y' + 4y = e^{5it}$! Ird fel az egyenlet egy megoldasat!

$y = Ae^{5it}$, $\left((5i)^2 + 3\cdot(5i) + 4\right)\cdot A e^{5it} = 1\cdot e^{5it}$

$$A = \frac{1}{-25+15i+4}, \quad y = \frac{e^{5it}}{-21+15i}$$

Legyen $2\partial^2_{xt} e^{i(kx-\omega t)} = 0$. Milyen algebrai egyenletet teljesit k es ω? Mi ennek a megoldashalmaza?

$2\cdot(ik)(-i\omega)e^{i(kx-\omega t)} = 0$, $k\omega = 0$

Megoldások halmaza:

$$\{k=0,\ \omega\in\mathbb{R}\} \cup \{k\in\mathbb{R},\ \omega=0\}$$

Legyen

$$\phi(0,x) = \sum_{n\in\mathbb{Z}} n^{-3}\frac{e^{inx}}{\sqrt{2\pi}}, \qquad \phi_t(0,x) = \sum_{n\in\mathbb{Z}} n^{-33}\frac{e^{inx}}{\sqrt{2\pi}},$$

$$\phi(t,x) = \sum_{n\in\mathbb{Z}} c_n(t)\frac{e^{inx}}{\sqrt{2\pi}}, \qquad \partial_{tt}\phi(t,x) = \frac{1}{4}\partial^2_{xx}\phi(t,x).$$

Ird fel a $c_n(t)$ fuggvenyekre vonatkozo kozonseges DE-ket (kezdeti feltetellel egyutt)!

$c_n''(t) = \frac{1}{4}\cdot(-n^2)\,c_n(t)$, $c_n(0) = n^{-3}$, $c_n'(0) = n^{-33}$

Mennyi $c_n(t)$? $c_n(t) = A_n\cdot\cos\left(\frac{n}{2}t\right) + B_n\cdot\sin\left(\frac{n}{2}t\right)$

$$c_n(t) = n^{-3}\cdot\cos\left(\frac{n}{2}t\right) + \frac{n^{-33}}{n/2}\sin\left(\frac{n}{2}t\right)$$

Mennyi

$$\exp\left[t\begin{pmatrix}-1 & 0\\ -2 & -1\end{pmatrix}\right] \ ?$$

Mivel kommutálnak

$$= \exp\left[\begin{pmatrix}-t & 0\\ 0 & -t\end{pmatrix} + \begin{pmatrix}0 & 0\\ -2t & 0\end{pmatrix}\right] = \exp\begin{pmatrix}-t & 0\\ 0 & -t\end{pmatrix}\cdot\exp\begin{pmatrix}0 & 0\\ -2t & 0\end{pmatrix}$$

$$= \begin{pmatrix}e^{-t} & 0\\ 0 & e^{-t}\end{pmatrix}\cdot\left[\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}0 & 0\\ -2t & 0\end{pmatrix} + \frac{1}{2}\underbrace{\begin{pmatrix}0 & 0\\ -2t & 0\end{pmatrix}^2}_{0} + \dots\right]$$

$$= \begin{pmatrix}e^{-t} & 0\\ 0 & e^{-t}\end{pmatrix}\begin{pmatrix}1 & 0\\ -2t & 1\end{pmatrix} = \begin{pmatrix}e^{-t} & 0\\ -2te^{-t} & e^{-t}\end{pmatrix}$$

PZh1, Diff.Egy., 2015.~~03.18.~~ 05.12 NEPTUN: Gyak.Vez.:

Név: Aláírás:

2. (3+4+3 pont)
a)

$$y' = f(x,y) = -y^2 - x^2;$$

Mennyi y'' ? Ird fel y masodrendu Talor polinomjat az $x = 3$ pont korul, ha $y(3) = 1$!

$$y'' = \left(\frac{\partial}{\partial x} + (-y^2-x^2)\frac{\partial}{\partial y}\right)(-y^2-x^2) = (-2x) + (-y^2-x^2)\cdot(-2y) = -2x + 2y^3 + 2x^2y$$

$y(3) = 1$

$y'(3) = -1^2 - 3^2 = -10$

$y''(3) = -2\cdot 3 + 2\cdot 1^3 + 2\cdot 3^2 \cdot 1$
$= 14$

$$y(3+\Delta x) \approx 1 - 10\Delta x + \frac{1}{2}\cdot 14\Delta x^2$$

b) Alkalmazd az Euler, illetve a Heun modszert a kovetkezo DE-re $\Delta x = 0.1$ lepeskozzel az $y(3) = 1$ kezdeti feltetel mellett!

$$y' = -y^2 - x^2.$$

Mit josol a ket modszer $y(3.1)$-re?

Euler:

$$y(3.1) \approx 1 + \underbrace{(-1^2-3^2)}_{-10}\cdot 0.1 = 0$$

Heun:

$$y(3.1) \approx 1 + \frac{1}{2}\Big((-10) + (-0^2 - 3.1^2)\Big)\cdot 0.1$$

c) Legyen $f(x) = 1/x^3$, $x_0 = 3$. Ird fel f-nek a linearis $f(x_0 + \Delta x) \approx T_1(x_0 + \Delta x)$ kozeliteset, ha $\Delta x = 0.1$! Mennyi $\max_{z\in[x_0,x_0+\Delta x]}|f''(z)|$? Adj nemtrivialis felso korlatot a kozelites $|\text{hiba}(\Delta x)| = |f(x_0 + \Delta x) - T_1(x_0 + \Delta x)|$ hibajara!

$f'(x) = -3x^{-4}$, $f''(x) = 12x^{-5}$

$f(3) = \frac{1}{27}$

$f'(3) = -\frac{1}{27}$

$f''(3) = \frac{4}{81}$

$$f(3.1) \approx \frac{1}{27} - \frac{1}{27}\cdot 0.1$$

$$|\text{hiba}(0.1)| \le \frac{1}{2}\cdot 0.1^2 \cdot \max_{z\in[3,3.1]} |f''(z)| =$$

$$= \frac{1}{2}\cdot 0.01 \cdot \frac{4}{81}$$

1

3a. (1+1+1+2 pont)

$y' = -y^2 + 4y$.

Keresd meg a DE fixpontjait!

$$-y^2+4y=0 \longrightarrow y_1=0\ ,\ y_2=4$$

Ird fel a fixpontok koruli linearizalt kozelito DE-t!

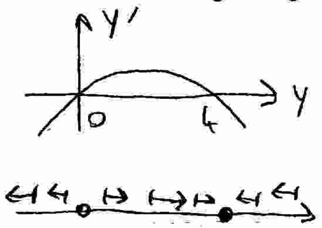
$$\frac{d(-y^2+4y)}{dy} = -2y+4$$

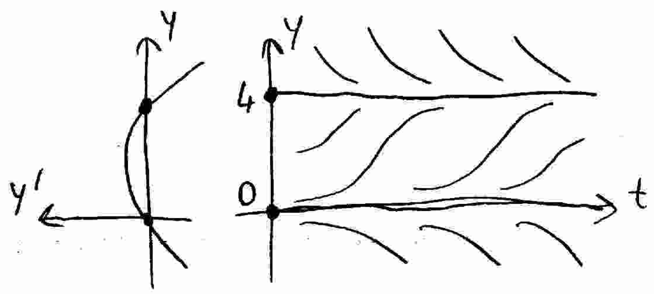
$$\frac{d}{dt}(y-0) = \frac{d}{dt}\Delta y_1 = (-2\cdot 0+4)\Delta y_1 = 4\Delta y_1\ ,\ \frac{d}{dt}(y-4) = \frac{d}{dt}\Delta y_2 = -4\Delta y_2$$

Ha $y(0) = 0.5$, mennyi

$\lim_{x\to\infty} y(x) =$ 4 $\qquad\qquad \lim_{x\to-\infty} y(x) =$ 0

Vazold a DE megoldasgorbeit!





3b. (2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} (y_2-3)y_1 \\ (y_2-5)(y_1-6). \end{pmatrix}$$

Keresd meg a DE fixpontjat!

$$\left.\begin{aligned}(y_2-3)y_1&=0\\(y_2-5)(y_1-6)&=0\end{aligned}\right\} \longrightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \text{ vagy } \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

Ird fel a fixpont koruli linearizalt kozelito DE-t!

$$Jac = \begin{pmatrix} \frac{\partial}{\partial y_1}\left[(y_2-3)y_1\right] & \frac{\partial}{\partial y_2}\left[(y_2-3)y_1\right] \\ \frac{\partial}{\partial y_1}\left[(y_2-5)(y_1-6)\right] & \frac{\partial}{\partial y_2}\left[(y_2-5)(y_1-6)\right] \end{pmatrix}$$

$$= \begin{pmatrix} y_2-3 & y_1 \\ y_2-5 & y_1-6 \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} y_1-0 \\ y_2-5 \end{pmatrix} = \frac{d}{dt}\overline{\Delta y_1} = \begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix}\Delta y_1 \quad\Bigg|\quad \frac{d}{dt}\begin{pmatrix} y_1-6 \\ y_2-3 \end{pmatrix} = \frac{d}{dt}\overline{\Delta y_2} = \begin{pmatrix} 0 & 6 \\ -2 & 0 \end{pmatrix}\Delta y_2$$