Diff. Eq. sample test 2.

The second test might contain variations of the exercises 3a, 4b, 5, 8, 9, 10, 12.

1. (a) Let $\left(\partial_{xx}^2 - \partial_{tt}^2 - 3\partial_{xt}^2\right) e^{i(kx-\omega t)} = 0$. What algebraic equation is satisfied by k and ω?
   Let $y'' - 3y' + y = e^{5it}$. Find a solution!

   (b) Let $\partial_{xt}^2 e^{i(kx+\omega t)} = 0$. What algebraic equation is satisfied by k and ω? Find the solutions!

2. (a)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 4y_1 \\ 4y_1 + 4y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

   How much is e^{tA} ? Find the particular solution of the DE!

   (b)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 5y_1 - 4y_2 \\ 4y_1 + 5y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

   Find the eigenvalues and eigenvectors of A ! Find the general solution of the DE! How much is e^{tA} ? Use e^{tA} to write down the particular solution!

3. (a) $y_t(t,x) = y_{xx}(t,x)$, $y(0,x) = \cos(7x)$. How much is $y(t,x)$?

   (b) $y_{tt}(t,x) = y_{xx}(t,x)$, $y(0,x) = \cos(7x)$, $y'(0,x) = \sin(5x)$. How much is $y(t,x)$?

4. (a) Let

$$\phi(0,x) = \sum_{n\in\mathbb{Z}} e^{-|n|} \frac{e^{inx}}{\sqrt{2\pi}}, \qquad \phi_t(0,x) = \sum_{n\in\mathbb{Z}} e^{-2|n|} \frac{1}{n^8} \frac{e^{inx}}{\sqrt{2\pi}},$$

$$\phi(t,x) = \sum_{n\in\mathbb{Z}} c_n(t) \frac{e^{inx}}{\sqrt{2\pi}}, \qquad \partial_{tt}\phi(t,x) = \frac{1}{4}\partial_{xx}^2 \phi(t,x).$$

   What ordinary differential equations are satisfied by the functions $c_n(t)$? (Do not forget the initial conditions!)

   (b) Legyen

$$\phi(0,x) = \sum_{n\in\mathbb{Z}} e^{-|n|} \frac{e^{inx}}{\sqrt{2\pi}}, \qquad \phi(t,x) = \sum_{n\in\mathbb{Z}} d_n(t) \frac{e^{inx}}{\sqrt{2\pi}}, \qquad \partial_t \phi(t,x) = \frac{1}{2}\partial_{xx}^2 \phi(t,x).$$

   What ordinary differential equations are satisfied by the functions $d_n(t)$? (Do not forget the initial conditions!)

5. (a)
   - Legyen

$$A = \begin{pmatrix} 2i & 1-2i \\ -i-1 & -3 \end{pmatrix}.$$

     How much is A^* ?
   - What is the inner product of $v = (2+3i, 4-i)^T$ and $w = (4,i)^T$?
   - Let $f_1 = (\sin(30°, i\cos(30°)^T$, $f_2 = (\cos(30°, z)^T$ be an orthonormed basis. How much is z ?
   - The vector $v = (5,6)^T$ can be expressed as $v = \alpha f_1 + \beta f_2$! How much is α ?

   (b)
   - Let $f(x) = \sin(4x) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, if $x \in (-\pi,\pi)$ How much is $\hat{f}_2$ es $\hat{f}_4$?
   - Legyen $f(x) = H(t-1)H(2-t) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, if $x \in (-\pi,\pi)$ How much is $\hat{f}_2$ es $\hat{f}_3$?
   - Let $f(x) = 1$, if $0 \le x \le \pi$, otherwise zero. If $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(p) e^{ipx}\, dx$, then how much is $\hat{f}(1)$?

6. (a)
   - Solve it! $y'' - 25y = 0,\ y(0) = 0,\ y'(0) = 1$.
   - Solve it! $G'' - 25G = \delta$, and $G(t) = 0$ for negative t.
   - Use G to write down the solution of $y'' - 25y = f(t)$, if $y(t) = f(t) = 0$ for negative t.

   (b)
   - Solve it! $y'' + 25y = 0,\ y(0) = 0,\ y'(0) = 1$.
   - Solve it! $G'' + 25G = \delta$, and $G(t) = 0$ for negativ t.
   - Use G to write down the solution of $y'' + 25y = f(t)$, if $y(t) = f(t) = 0$ for negative t.

7. (a)
   - Solve it! $y' - 25y = 0,\ y(0) = 0$.
   - Solve it! $G' - 25G = \delta$, and $G(t) = 0$ for negative t.
   - Use G to write down the solution of $y' - 25y = f(t)$, if $y(t) = f(t) = 0$ for negative t.

8. (a)
   - Use the definition of the Laplace transform for the computation of $\mathcal{L}(\sin(2t))$.
   - Use the definition of the Laplace transform for the computation of $\mathcal{L}(H(-t+4)e^{-5t})$.
   - Let $f(t) = H(t-3)$ and $g(t) = e^{5t}$. Compute the $h = f * g$ convolution!
   - How much is $\mathcal{L}(f(t))\mathcal{L}(g(t)) - \mathcal{L}(h(t))$?

   (b)
   - se the definition of the Laplace transform for the computation of $\mathcal{L}(tH(t-2))$.
   - se the definition of the Laplace transform for the computation of $.\mathcal{L}(H(-t-4)e^{-5t})$
   - Let $f(t) = \sin(t)$ and $g(t) = e^{5t}$. Compute the $h = f * g$ konvolution!
   - How much is $\mathcal{L}(f(t))\mathcal{L}(g(t)) - \mathcal{L}(h(t))$?

9. (a) $y' - 4y = 3,\ y(0) = 2$.
   - Compute the $Y(s)$ Laplace transform of $y(t)$!
   - Compute the partial fraction decomposition of $Y(s)$!
   - How much is $\mathcal{L}^{-1}(Y(s))$?
   - How much is $y(t)$?

10. (a) $y'' - 4y = 3,\ y(0) = 2,\ y'(0) = 20$
   - Compute the $Y(s)$ Laplace transform of $y(t)$!
   - Compute the partial fraction decomposition of $Y(s)$!
   - How much is $y(t)$?

   (b) $y'' + 4y = 3,\ y(0) = 2,\ y'(0) = 20$
   - Compute the $Y(s)$ Laplace transform of $y(t)$!
   - Compute the partial fraction decomposition of $Y(s)$!
   - How much is $y(t)$?

11. (a) Solve it! $y'' - 25y = 0,\ y(0) = 10,\ y'(0) = 20$.

   (b) Solvi it with the method of the variation of the constant! $y' + 3y = 5$.

12. (a) Let

$$\frac{d}{dt}\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} + \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = \begin{pmatrix} 1+t \\ t+t^2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

   How much is

$$\begin{pmatrix} Y_1(s) \\ Y_2(s) \end{pmatrix} \quad ?$$

   (Do not compute the inverse matrix!)

13. (a) Let

$$\frac{d}{dt}\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}, \qquad f_1(t) = f_2(t) = y_1(t) = y_2(t) = 0,\ \text{ha } t < 0.$$

   Find the fundamental solution and use it express the solution of the DE!