4. (3+2+3+2)
Let

$$\phi(0,x)=\sum_{n\in\mathbb{Z}} n^{-2}\sin(n)\frac{e^{inx}}{\sqrt{2\pi}},\qquad \phi(t,x)=\sum_{n\in\mathbb{Z}} c_n(t)\frac{e^{inx}}{\sqrt{2\pi}},\qquad \partial_t\phi(t,x)=6\partial^2_{xx}\phi(t,x).$$

What ordinary differential equations are satisfied by the functions $c_n(t)$? (Do not forget the initial conditions!)

Compute $c_5(6)$!

Let $(f,g)=\int_0^\pi \bar{f}(x)g(x)\,dx$.
Compute $(\sin(x),\sin(2x))$!

Compute $(\sin(x),\cos(x))$!

B Test 2, Diff.Eq., 2015.05.04. NEPTUN: Name:

1. (2+1+2+3+1+1)
Use the definition of the Laplace tr. for the computation of $F(s)=\mathcal{L}(f(t))=\mathcal{L}(e^{5t-7})$.
$F(s)=$

For what values of s does the improper integral exist?

$F(s)=\mathcal{L}(f(t))=\mathcal{L}(H(-t-4)e^{-5t})$ (Here H is the Heaviside function.)
$F(s)=$

Compute the $h=f*g$ convolution of $f(t)=4t$ and $g(t)=3$!

Compute the $h=g*f$ convolution of $f(t)=4t$ and $g(t)=3$!

How much is $\mathcal{L}(f(t))\mathcal{L}(g(t))-\mathcal{L}(h(t))$?

2. (2+2+3+3)
Let $f_1 = (i/\sqrt{2}, i/\sqrt{2})^T$, $f_2 = (1/\sqrt{2}, z)^T$ be an orthonormal basis of $\mathbb{C}^2$. How much is z?

The vector $v = (7, 8)^T$ can be expressed as a linear combination $v = \alpha f_1 + \beta f_2$! Compute α !

Let $f(x) = H(t)H(-t + \pi/2) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, if $x \in (-\pi, \pi)$ Compute $\hat{f}_5$!

Express $\hat{f}_{-5} \frac{e^{i(-5)x}}{\sqrt{2\pi}} + \hat{f}_5 \frac{e^{i5x}}{\sqrt{2\pi}}$ with the help of trigonometric functions!

3. $(3 + 2 + 1 + 4)$
$y'' - 4y = (t + 1)^2,\ y(0) = 6,\ y'(0) = 7$. How much is $Y(s)$? $(\mathcal{L}(t^n) = \frac{n!}{s^{n+1}})$
$Y(s) =$

Write down the partial fraction decomposition of $Y(s)$! (Do not compute the coefficients!)

How much is $y(t)$?

Let

$$\frac{d}{dt}\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} + \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = \begin{pmatrix} 1+t \\ t+t^2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

Compute

$$\begin{pmatrix} Y_1(s) \\ Y_2(s) \end{pmatrix}$$

(Do not compute the inverse matrix!)