B Zh2, Diff.Egy., 2015.05.04. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. (2+1+3+3+1 pont)
Szamitsd ki a Laplace tr. definicioja alapjan a kovetkezoket:
a) $F(s) = \mathcal{L}(f(t)) = \mathcal{L}(e^{5t-7})$.
$F(s)=$

$$\int_0^{\infty} e^{-st} \cdot e^{5t-7} dt = e^{-7} \int_0^{\infty} e^{-(s-5)t} dt$$

$$= e^{-7} \cdot \left[\frac{e^{-(s-5)t}}{-(s-5)} \right]_0^{\infty} = e^{-7} \left[0 - \frac{1}{-(s-5)} \right] = \frac{e^{-7}}{s-5}$$

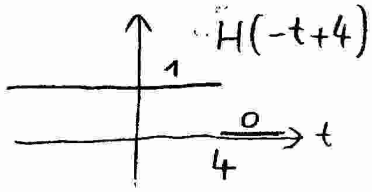
Esetunkben milyen s eseten letezik a Laplace transzformaciot definialo impropius integral?

$$\mathrm{Re}\, s > 5$$

$F(s) = \mathcal{L}(f(t)) = \mathcal{L}(H(-t+4)e^{-5t})$ (Itt H a Heaviside fuggveny.)
$F(s)=$

$$\int_0^{\infty} e^{-st} H(-t+4) e^{-5t} dt = \int_0^{4} e^{-(s+5)t} =$$

$$= \left[\frac{e^{-(s+5)t}}{-(s+5)} \right]_0^{4} = \frac{1}{s+5} - \frac{e^{-(s+5)\cdot 4}}{s+5}$$



b) Szamold ki az $f(t) = 4t$ es a $g(t) = -5t$ fuggvenyek $h = f * g$ konvoluciojat!

$$(f*g)(t) = \int_0^{t} 4(t-\tau) \cdot (-5\tau) d\tau = -20 \int_0^{t} t\tau - \tau^2 d\tau$$

$$= -20 \left[t \frac{\tau^2}{2} - \frac{\tau^3}{3} \right]_0^{t} = -20 \cdot \left(\frac{t^3}{2} - \frac{t^3}{3} \right) = -\frac{20}{6} t^3 = -\frac{10}{3} t^3$$

Mennyi $\mathcal{L}(f(t))\mathcal{L}(g(t)) - \mathcal{L}(h(t))$?

$$= 0$$

2. (1+1+2+3+3 pont)
Legyen

$$A = \begin{pmatrix} i & 1-2i \\ -i & 3 \end{pmatrix}.$$

Mennyi A^* ?

$(A^*)_{i\gamma} = \overline{A}_{\gamma i}$ $\qquad$ $A^* = \begin{pmatrix} -i & i \\ 1+2i & 3 \end{pmatrix}$

Legyen $f_1 = (1/\sqrt{2}, i/\sqrt{2})^T$, $f_2 = (i/\sqrt{2}, z)^T$ egy ortonormalt bazis. Mennyi z ?

$$(f_1, f_2) = 0 = \overline{\frac{1}{\sqrt{2}}} \cdot \frac{i}{\sqrt{2}} + \overline{\frac{i}{\sqrt{2}}} \cdot z = \frac{i}{2} - \frac{i}{\sqrt{2}} z = 0 \longrightarrow z = \frac{1}{\sqrt{2}}$$

A $v = (7,8)^T$ vektor kifejezheto az f-ek linearis $\alpha f_1 + \beta f_2$ kombinaciojakent! Mennyi α ?

$$\alpha = (f_1, v) = \overline{\frac{1}{\sqrt{2}}} \cdot 7 + \overline{\frac{i}{\sqrt{2}}} \cdot 8 = \frac{7}{\sqrt{2}} - \frac{8i}{\sqrt{2}} = \frac{7}{\sqrt{2}} - 4\sqrt{2}\, i$$

Legyen $f(x) = 5H(\cancel{x}) \overset{5H(x)}{=} \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, ha $x \in (-\pi, \pi)$ Mennyi $\hat{f}_{-5}$?

$$\hat{f}_{-5} = \left(\frac{e^{i(-5)x}}{\sqrt{2\pi}}, 5H(x) \right) = \int_{-\pi}^{\pi} \frac{e^{-5ix}}{\sqrt{2\pi}} 5H(x)\,dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\pi} e^{5ix} \cdot 5\,dx = \frac{5}{\sqrt{2\pi}} \left[\frac{e^{5ix}}{5i} \right]_0^{\pi} = \frac{1}{\sqrt{2\pi}} \left(\frac{\overset{-1}{e^{5i\pi}}}{i} - \frac{1}{i} \right) = +\frac{i\sqrt{2}}{\sqrt{\pi}}$$

Fejezd ki trigonometrikus fuggvenyek segitsegevel $\hat{f}_{-5}\frac{e^{i(-5)x}}{\sqrt{2\pi}} + \hat{f}_5 \frac{e^{i5x}}{\sqrt{2\pi}}$-t!

$$\hat{f}_5 = -\frac{i\sqrt{2}}{\sqrt{\pi}}, \qquad + \frac{i\sqrt{2}}{\sqrt{\pi}} \frac{e^{-5ix}}{\sqrt{2\pi}} - \frac{i\sqrt{2}}{\sqrt{\pi}} \frac{e^{5ix}}{\sqrt{2\pi}} =$$

$$= \frac{-i}{\pi} 2i \cdot \frac{e^{5ix} - e^{-5ix}}{2i} = +\frac{2}{\pi} \sin(5x)$$

előző számításban
$-5 \longrightarrow 5$,
vagy mivel f valós,
így $\hat{f}_5 = \overline{\hat{f}_{-5}}$

3. (5 × 2 pont)
$y'' + 9y = 5t^3$, $y(0) = 6$, $y'(0) = 7$. Mennyi $Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$)
$Y(s) =$

$$\left(s^2 Y(s) - s \cdot 6 - 7\right) + 9\, Y(s) = 5 \cdot \frac{3!}{s^4}$$

$$Y(s) = \frac{1}{s^2+9}\left(5 \cdot \frac{3!}{s^4} + 6 \cdot s + 7\right)$$

Mi a megoldasa a $y'' + 9y = 0$, $y(0) = 6$, $y'(0) = 7$ DE-nek?

$$y = e^{\lambda t} \longrightarrow \lambda^2 + 9 = 0, \quad \lambda_1 = 3i, \quad \lambda_2 = -3i$$

$$y_{ált} = C_1 \cos(3x) + C_2 \sin(3x) \qquad C_1 \cdot 1 + C_2 \cdot 0 = 6$$

$$y'_{ált} = -3C_1 \sin(3x) + C_2 \cdot 3\cos(3x) \qquad -3C_1 \cdot 0 + 3C_2 \cdot 1 = 7$$

$$y_{part} = 6 \cdot \cos(3x) + \frac{7}{3} \sin(3x)$$

Oldd meg a $G'' - 9G = \delta(t)$ egyenletet, ahol $G(t) = 0$, ha $t < 0$!

$$t \approx 0: \quad G \approx F, \text{ ahol } F'' = \delta \longrightarrow F(t) = \begin{cases} 0, & \text{ha } t<0 \\ t, & \text{ha } t>0 \end{cases}$$

tehát $G(0^+) - G(0^-) = F(0^+) - F(0^-) = 0$

$$G'(0^+) - G'(0^-) = F'(0^+) - F'(0^-) = 1$$

$t>0$: $G(t)$ megoldása a $G'' - 9G = 0$, $G(0) = 0$, $G'(0) = 1$ egyenletnek

$$\downarrow$$

$$G(t) = C_1 e^{3t} + C_2 e^{-3t} \qquad C_1 + C_2 = 0$$

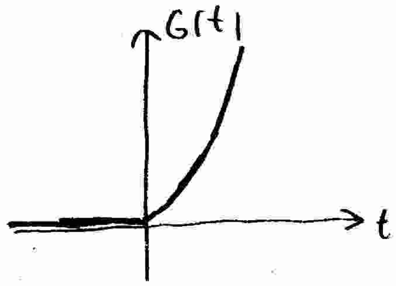
$$G' = 3C_1 e^{3t} - 3C_2 e^{-3t} \qquad 3C_1 - 3C_2 = 1$$

$$G(t) = \frac{1}{6}\left(e^{3t} - e^{-3t}\right), \text{ ha } t > 0$$

Rajzold le $G(t)$-t!

Tehát

$$G(t) = \begin{cases} 0, & \text{ha } t<0 \\ \frac{1}{6}\left(e^{3t} - e^{-3t}\right), & \text{ha } t>0 \end{cases}$$



Ird fel a $y'' - 9y = f(t)$ egyenlet megoldasat, ha $y(t) = f(t) = 0$ amikor $t < 0$!

$$y(t) = \int_{-\infty}^{\infty} G(t-\tau) f(\tau)\, d\tau = \int_{-\infty}^{t} \frac{1}{6}\left(e^{3(t-\tau)} + e^{-3(t-\tau)}\right) f(\tau)\, d\tau$$

G retardált Green függvény

$$\left(= \int_{0}^{t} \frac{1}{6}\left(e^{3(t-\tau)} - e^{-3(t-\tau)}\right) d\tau \right)$$

$f(t) = y(t) = 0$, ha $t < 0$

4. (2+2+3+3 pont)

Legyen $(3\partial^2_{x\dot{x}} - 7\partial^2_{xx} + \partial^2_{xt})\, e^{i(kx+\omega t)} = 0$. Milyen algebrai egyenletet teljesit k es ω?

$$3\cdot(ik)^2 - 7(ik)^2 + (ik)(i\omega) = 0$$

$$-3k^2 + 7k^2 - k\omega = 4k^2 - k\omega = 0$$

Legyen

$$\phi(0,x) = \sum_{n\in\mathbb{Z}} n^{-2}\frac{e^{inx}}{\sqrt{2\pi}}, \qquad \phi(t,x) = \sum_{n\in\mathbb{Z}} c_n(t)\frac{e^{inx}}{\sqrt{2\pi}}, \qquad \partial_t\phi(t,x) = 6\partial^2_{xx}\phi(t,x).$$

Ird fel a $c_n(t)$ fuggvenyekre vonatkozo kozonseges DE-ket (kezdeti feltetellel egyutt)!

$$\partial_t\varphi = \sum c_n'(t)\frac{e^{inx}}{\sqrt{2\pi}}, \quad \partial^2_{xx}\varphi = \sum c_n(t)(+in)^2\cdot\frac{e^{inx}}{\sqrt{2\pi}}$$

$$6c_n'(t) = -n^2 c_n(t), \quad c(0) = n^{-2}$$

Legyen

$$\phi(0,x) = \sum_{n\in\mathbb{Z}} n^{-2}\frac{e^{inx}}{\sqrt{2\pi}}, \qquad \phi_t(0,x) = \sum_{n\in\mathbb{Z}} n^{-4}\frac{e^{inx}}{\sqrt{2\pi}},$$

$$\phi(t,x) = \sum_{n\in\mathbb{Z}} c_n(t)\frac{e^{inx}}{\sqrt{2\pi}}, \qquad \partial_{tt}\phi(t,x) = 6\partial^2_{xx}\phi(t,x).$$

Ird fel a $c_n(t)$ fuggvenyekre vonatkozo kozonseges DE-ket (kezdeti feltetellel egyutt)!

$$\partial^2_{tt}\varphi = \sum c_n''(t)\frac{e^{inx}}{\sqrt{2\pi}}, \quad \partial^2_{xx} = \sum c_n(t)(in)^2\cdot\frac{e^{inx}}{\sqrt{2\pi}}$$

$$c_n''(t) = 6\cdot(-n^2)c_n(t), \quad c_n(0) = n^{-2}, \quad c_n'(0) = n^{-4}$$

Mennyi $\exp\left[t\begin{pmatrix}5 & 6\\ 0 & 5\end{pmatrix}\right]$

Mivel $\begin{pmatrix}5t & 0\\ 0 & 5t\end{pmatrix}$ és $\begin{pmatrix}0 & 6t\\ 0 & 0\end{pmatrix}$ kommutál egymással

$$e^{\begin{pmatrix}5t & 6t\\ 0 & 5t\end{pmatrix}} = e^{\begin{pmatrix}5t & 0\\ 0 & 5t\end{pmatrix}+\begin{pmatrix}0 & 6t\\ 0 & 0\end{pmatrix}} = e^{\begin{pmatrix}5t & 0\\ 0 & 5t\end{pmatrix}}\cdot e^{\begin{pmatrix}0 & 6t\\ 0 & 0\end{pmatrix}}$$

$$= \begin{pmatrix}e^{5t} & 0\\ 0 & e^{5t}\end{pmatrix}\left[\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}0 & 6t\\ 0 & 0\end{pmatrix} + \frac{1}{2}\underbrace{\begin{pmatrix}0 & 6t\\ 0 & 0\end{pmatrix}^2}_{=0} + \dots\right]$$

$$= \begin{pmatrix}e^{5t} & 0\\ 0 & e^{5t}\end{pmatrix}\begin{pmatrix}1 & 6t\\ 0 & 1\end{pmatrix} = \begin{pmatrix}e^{5t} & 6te^{5t}\\ 0 & e^{5t}\end{pmatrix}$$

A Zh2, Diff.Egy., 2015.05.04. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1.(1+1+2+3+3 pont)
Legyen

$$A = \begin{pmatrix} 2 & -2i \\ -i+1 & 3i \end{pmatrix}.$$

Mennyi A^* ?

$$(A^*)_{ij} = \overline{A_{ji}} \qquad A^* = \begin{pmatrix} 2 & i+1 \\ 2i & -3i \end{pmatrix}$$

Legyen $f_1 = (1/\sqrt{2}, i/\sqrt{2})^T$, $f_2 = (z, i/\sqrt{2})^T$ egy ortonormalt bazis. Mennyi z ?

$$(f_1, f_2) = \overline{\frac{1}{\sqrt{2}}} \cdot z + \overline{\frac{i}{\sqrt{2}}} \cdot \frac{i}{\sqrt{2}} = \frac{z}{\sqrt{2}} + \frac{1}{2} = 0 \to z = -\frac{1}{\sqrt{2}}$$

A $v = (5,6)^T$ vektor kifejezheto az f-ek linearis $\alpha f_1 + \beta f_2$ kombinaciojakent! Mennyi α ?

$$\alpha = (f_1, v) = \overline{\frac{1}{\sqrt{2}}} \cdot 5 + \overline{\frac{i}{\sqrt{2}}} \cdot 6 = \frac{5}{\sqrt{2}} - \frac{6}{\sqrt{2}} i = \frac{5}{\sqrt{2}} - 3\sqrt{2} \cdot i$$

$-H(-x)$

Legyen $f(x) = -H(\cancel{x}) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, ha $x \in (-\pi, \pi)$ Mennyi $\hat{f}_{-5}$?

$$\hat{f}_{-5} = \left(\frac{e^{i(-5)x}}{\sqrt{2\pi}}, -H(-x) \right) = \int_{-\pi}^{\pi} \overline{\frac{e^{-5ix}}{\sqrt{2\pi}}} \cdot (-H(-x))\, dx$$

-1

$\downarrow$

$e^{5i(-\pi)}$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{0} e^{5ix} \cdot (-1)\, dx = \frac{1}{\sqrt{2\pi}} \left(-\left[\frac{e^{5ix}}{5i} \right]_{-\pi}^{0} \right) = -\frac{1}{\sqrt{2\pi}} \left(\frac{1}{5i} - \frac{e^{5i(-\pi)}}{5i} \right) = \frac{+i \cdot \sqrt{2}}{5\sqrt{\pi}}$$

Fejezd ki trigonometrikus fuggvenyek segitsegevel $\hat{f}_{-5}\frac{e^{i(-5)x}}{\sqrt{2\pi}} + \hat{f}_5 \frac{e^{i5x}}{\sqrt{2\pi}}$-t!

$$\hat{f}_5 = -\frac{i \cdot \sqrt{2}}{5\sqrt{\pi}}, \qquad + \frac{i\sqrt{2}}{5\sqrt{\pi}} \frac{e^{-5ix}}{\sqrt{2\pi}} - \frac{i\sqrt{2}}{5\sqrt{\pi}} \frac{e^{5ix}}{\sqrt{2\pi}} = \frac{-i\sqrt{2}}{5\sqrt{\pi}\cdot\sqrt{2\pi}} \cdot 2i \cdot \frac{e^{5ix} - e^{-5ix}}{2i}$$

$$= \frac{+2}{5\pi} \cdot \sin(5x)$$

$\uparrow$

előző számításban
$-5 \longrightarrow 5$,
vagy mivel f valós,
így $\hat{f}_5 = \hat{f}_{-5}$

1

2. (2+1+3+3+1 pont)
Szamitsd ki a Laplace tr. definicioja alapjan a kovetkezoket:
a) $F(s) = \mathcal{L}(f(t)) = \mathcal{L}(e^{-5t+7})$.

$F(s)=$ $\int_0^\infty e^{-st} \cdot e^{-5t+7} \, dt = e^7 \cdot \int_0^\infty e^{-(s+5)t} \, dt = e^7 \cdot \left[\frac{e^{-(s+5)t}}{-(s+5)} \right]_0^\infty$

$= e^7 \cdot \left[0 - \frac{1}{-(s+5)} \right] = \frac{e^7}{s+5}$

Esetunkben milyen s eseten letezik a Laplace transzformaciot definialo impropius integral?

$\mathrm{Re}\, s > -5$

$F(s) = \mathcal{L}(f(t)) = \mathcal{L}(H(t-4)e^{5t})$ (Itt H a Heaviside fuggveny.)

$F(s)=$ $\int_0^\infty e^{-st} H(t-4) e^{5t} \, dt = \int_4^\infty e^{-(s-5)t} \, dt$ $\quad H(t-4) = 0$ ha $t \in [0,4)$, amúgy $=1$

$= \left[\frac{e^{-(s-5)t}}{-(s-5)} \right]_4^\infty = 0 - \frac{e^{-(s-5)\cdot 4}}{-(s-5)} = \frac{e^{-(s-5)\cdot 4}}{s-5}$

b) Szamold ki az $f(t) = e^{4t}$ es a $g(t) = e^{-5t}$ fuggvenyek $h = f * g$ konvoluciojat!

$(f * g)(t) = \int_0^t e^{4(t-\tau)} \cdot e^{-5\tau} \, d\tau = e^{4t} \cdot \int_0^t e^{-9\tau} \, d\tau$

$= e^{4t} \left[\frac{e^{-9\tau}}{-9} \right]_0^t = e^{4t} \left(\frac{e^{-9t}}{-9} - \frac{1}{-9} \right) = \frac{1}{9} \left(e^{4t} - e^{-5t} \right)$

Mennyi $\mathcal{L}(f(t))\mathcal{L}(g(t)) - \mathcal{L}(h(t))$?

$= 0$

3. (5 × 2 pont)
$y'' - 9y = 5t^3$, $y(0) = 6$, $y'(0) = 7$. Mennyi $Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$)
$Y(s) =$

$$\left(s^2 Y(s) - 6s - 7\right) - 9Y(s) = 5 \cdot \frac{3!}{s^4}$$

$$Y(s) = \frac{1}{s^2 - 9}\left(5 \cdot \frac{3!}{s^4} + 6s + 7\right)$$

Mi a megoldasa a $y'' - 9y = 0$, $y(0) = 6$, $y'(0) = 7$ DE-nek?

$$y = e^{\lambda t} \rightarrow \lambda^2 - 9 = 0\,;\ \lambda_1 = 3,\ \lambda_2 = -3$$

$$y_{ált} = C_1 e^{3t} + C_2 e^{-3t} \qquad C_1 + C_2 = 6$$

$$y'_{ált} = 3C_1 e^{3t} - 3C_2 e^{-3t} \qquad 3C_1 - 3C_2 = 7 \quad \rightarrow C_1 = \frac{25}{6},\ C_2 = \frac{11}{6}$$

$$y_{part} = \frac{25}{6} e^{3t} + \frac{11}{6} e^{-3t}$$

Oldd meg a $G'' + 9G = \delta(t)$ egyenletet, ahol $G(t) = 0$, ha $t < 0$!

$$t \approx 0: \quad G \approx F,\ \text{ahol}\ F'' = \delta \longrightarrow F(t) = \begin{cases} 0, & \text{ha } t < 0 \\ t, & \text{ha } t > 0 \end{cases}$$

$$\text{tehát} \quad G(0^+) - G(0^-) = F(0^+) - F(0^-) = 0$$

$$G'(0^+) - G'(0^-) = F(0^+) - F(0^-) = 1$$

$t > 0$: $G(t)$ megoldása a $G'' + 9G = 0$, $G(0) = 0$, $G'(0) = 1$ DE-nek

$$G(t) = C_1 \cos(3t) + C_2 \sin(3t) \qquad C_1 \cdot 1 + C_2 \cdot 0 = 0$$

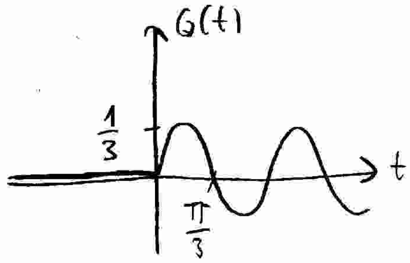
$$G'(t) = -C_1 \cdot 3 \sin(3t) + 3C_2 \cos(3t) \qquad -3C_1 \cdot 0 + 3C_2 \cdot 1 = 1$$

$$G(t) = \frac{1}{3}\sin(3t),\ \text{ha}\ t > 0$$

Rajzold le $G(t)$-t!

Tehát

$$G(t) = \begin{cases} 0, & \text{ha } t < 0 \\ \frac{1}{3}\sin(3t), & \text{ha } t > 0 \end{cases}$$



Ird fel $G(t)$ segitsegevel az $y'' + 9y = f(t)$ egyenlet megoldasat, ha $y(t) = f(t) = 0$ amikor $t < 0$!

$$y(t) = \int_{-\infty}^{\infty} G(t-\tau) f(\tau)\, d\tau = \int_{-\infty}^{t} \frac{1}{3}\sin(3 \cdot (t-\tau))\, f(\tau)\, d\tau$$

$G(t)$ a retardált Green függvény

$$\left(= \int_{0}^{t} \frac{1}{3}\sin(3 \cdot (t-\tau))\, f(\tau)\, d\tau \right)$$

$f(t) = y(t) = 0$, ha $t < 0$

4.(2+2+3+3 pont)
Legyen $(3\partial^2_{xx} - 7\partial^2_{xx} - 2\partial^2_{xt})\, e^{i(kx-\omega t)} = 0$. Milyen algebrai egyenletet teljesit k es ω?

$$3\cdot(ik)^2 - 7(ik)^2 - 2(ik)(-i\omega) = 0$$

$$-3k^2 + 7k^2 - 2k\omega = 4k^2 - 2k\omega = 0$$

Legyen

$$\phi(0,x) = \sum_{n\in\mathbb{Z}} \frac{1}{n^6}\frac{e^{inx}}{\sqrt{2\pi}}, \qquad \phi_t(0,x) = \sum_{n\in\mathbb{Z}} \frac{1}{n^8}\frac{e^{inx}}{\sqrt{2\pi}},$$

$$\phi(t,x) = \sum_{n\in\mathbb{Z}} c_n(t)\frac{e^{inx}}{\sqrt{2\pi}}, \qquad 6\partial_{tt}\phi(t,x) = \partial^2_{xx}\phi(t,x).$$

Ird fel a $c_n(t)$ fuggvenyekre vonatkozo kozonseges DE-ket (kezdeti feltetellel egyutt)!

$$\partial^2_{tt}\varphi = \sum c_n''(t)\frac{e^{inx}}{\sqrt{2\pi}}, \quad \partial_{xx}\varphi = \sum c_n(t)(+in)^2\cdot\frac{e^{inx}}{\sqrt{2\pi}}$$

$$6\cdot c_n''(t) = -n^2 c_n(t), \quad c_n(0) = \frac{1}{n^6}, \quad c_n'(t) = \frac{1}{n^8}$$

Legyen

$$\cancel{\phi(0,x) =} \sum_{n\in\mathbb{Z}} \frac{1}{n}\frac{e^{inx}}{\sqrt{2\pi}}, \qquad \phi(t,x) = \sum_{n\in\mathbb{Z}} d_n(t)\frac{e^{inx}}{\sqrt{2\pi}}, \qquad 6\partial_t\phi(t,x) = \partial^2_{xx}\phi(t,x).$$

Ird fel a $d_n(t)$ fuggvenyekre vonatkozo kozonseges DE-ket (kezdeti feltetellel egyutt)!

$$6\cdot d_n'(t) = -n^2 d_n(t), \quad d_n(0) = \frac{1}{n}$$

Mennyi

$$\exp\left[t\begin{pmatrix}-4 & 0\\ 6 & -4\end{pmatrix}\right]$$

Mivel $\begin{pmatrix}-4t & 0\\ 0 & -4t\end{pmatrix}\begin{pmatrix}0 & 0\\ 6t & 0\end{pmatrix} = \begin{pmatrix}0 & 0\\ 6t & 0\end{pmatrix}\begin{pmatrix}-4t & 0\\ 0 & -4t\end{pmatrix}$

$$e^{\begin{pmatrix}-4t & 0\\ 6t & -4t\end{pmatrix}} = e^{\begin{pmatrix}-4t & 0\\ 0 & -4t\end{pmatrix}}\cdot e^{\begin{pmatrix}0 & 0\\ 6t & 0\end{pmatrix}} =$$

$$= \begin{pmatrix}e^{-4t} & 0\\ 0 & e^{-4t}\end{pmatrix}\cdot\left[\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}0 & 0\\ 6t & 0\end{pmatrix} + \frac{1}{2}\underbrace{\begin{pmatrix}0 & 0\\ 6t & 0\end{pmatrix}^2}_{=0} + \dots\right]$$

$$= \begin{pmatrix}e^{-4t} & 0\\ 0 & e^{-4t}\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ 6t & 1\end{pmatrix} = \begin{pmatrix}e^{-4t} & 0\\ 6te^{-4t} & e^{-4t}\end{pmatrix}$$