PZh1, Diff.Egy., 2015.~~03.18.~~ 05.12 NEPTUN: Gyak.Vez.:

Név: Aláírás:

2. (3+4+3 pont)

a)

$$y' = f(x, y) = -y^2 - x^2;$$

Mennyi y'' ? Ird fel y masodrendu Talor polinomjat az $x = 3$ pont korul, ha $y(3) = 1$!

$$y'' = \left(\frac{\partial}{\partial x} + (-y^2-x^2)\frac{\partial}{\partial y}\right)(-y^2-x^2) = (-2x) + (-y^2-x^2)\cdot(-2y) = -2x + 2y^3 + 2x^2y$$

$y(3) = 1$

$y'(3) = -1^2 - 3^2 = -10$

$y''(3) = -2\cdot 3 + 2\cdot 1^3 + 2\cdot 3^2\cdot 1$

$= 14$

$$y(3+\Delta x) \approx 1 - 10\Delta x + \frac{1}{2}\cdot 14\Delta x^2$$

b) Alkalmazd az Euler, illetve a Heun modszert a kovetkezo DE-re $\Delta x = 0.1$ lepeskozzel az $y(3) = 1$ kezdeti feltetel mellett!

$$y' = -y^2 - x^2.$$

Mit josol a ket modszer $y(3.1)$-re?

Euler:

$$y(3.1) \approx 1 + \underbrace{(-1^2 - 3^2)}_{-10}\cdot 0.1 = 0$$

Heun:

$$y(3.1) \approx 1 + \frac{1}{2}\left((-10) + (-0^2 - 3.1^2)\right)\cdot 0.1$$

c) Legyen $f(x) = 1/x^3$, $x_0 = 3$. Ird fel f-nek a linearis $f(x_0 + \Delta x) \approx T_1(x_0 + \Delta x)$ kozeliteset, ha $\Delta x = 0.1$! Mennyi $\max_{z\in[x_0,x_0+\Delta x]}|f''(z)|$? Adj nemtrivialis felso korlatot a kozelites $|\text{hiba}(\Delta x)| = |f(x_0 + \Delta x) - T_1(x_0 + \Delta x)|$ hibajara!

$f'(x) = -3x^{-4}$, $f''(x) = 12x^{-5}$

$f(3) = \frac{1}{27}$ $\quad f(3.1) \approx \frac{1}{27} - \frac{1}{27}\cdot 0.1$

$f'(3) = -\frac{1}{27}$

$f''(3) = \frac{4}{81}$

$$|\text{hiba}(0.1)| \le \frac{1}{2}\cdot 0.1^2 \cdot \max_{z\in[3,3.1]}|f''(z)| =$$

$$= \frac{1}{2}\cdot 0.01\cdot\frac{4}{81}$$

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1a. (1+1+1+2 pont)

$y' = -y^3 + \cancel{4x} + 4y = -y(y^2-4) = -y(y+2)(y-2)\,, \quad \frac{d(-y^3+4y)}{dy} = -3y^2+4 = f'(y)$

Keresd meg a DE fixpontjait!

$-y^3+4y=0 \longrightarrow y_1 = 0,\ y_2=-2,\ y_3=2$

Ird fel a fixpontok koruli linearizalt kozelito DE-t!

$y_1=0$: $f'(0) = -3\cdot 0^2+4$ $\quad\frac{d}{dx}(y-0) = \frac{d}{dx}\Delta y = 4\cdot\Delta y$

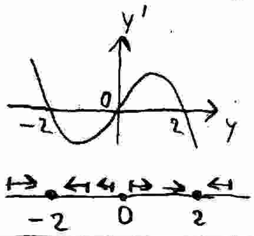
$y_2=-2$ $\quad\frac{d}{dx}(y-(-2)) = \frac{d}{dx}\Delta y = -8$

$y_2 = 2$ $\quad\frac{d}{dx}(y-2) = \frac{d}{dx}\Delta y = -8\Delta y$

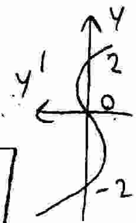
Ha $y(0) = 0.5$, mennyi

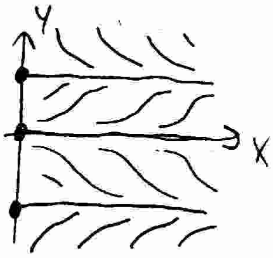
$\lim_{x\to\infty} y(x) =$ 2

$\lim_{x\to-\infty} y(x) =$ 0



Vazold a DE megoldasgorbeit!





1b. (2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} (y_2-3)(y_1-4) \\ (y_2-5)(y_1-6). \end{pmatrix}$$

Keresd meg a DE fixpontjat!

$$\left.\begin{matrix} (y_2-3)(y_1-4)=0 \\ (y_2-5)(y_1-6)=0 \end{matrix}\right\} \longrightarrow \underbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}}_{P_1} \text{ vagy } \underbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}}_{P_2}$$

Ird fel a fixpont koruli linearizalt kozelito DE-t!

$$\mathrm{Jac} = \begin{pmatrix} \frac{\partial}{\partial y_1}(y_2-3)(y_1-4) & \frac{\partial}{\partial y_2}(y_2-3)(y_1-4) \\ \frac{\partial}{\partial y_1}(y_2-5)(y_1-6) & \frac{\partial}{\partial y_2}(y_2-5)(y_1-6) \end{pmatrix} = \begin{pmatrix} y_2-3 & y_1-4 \\ y_2-5 & y_1-6 \end{pmatrix}$$

$$\mathrm{Jac}(P_1) = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \qquad \mathrm{Jac}(P_2) = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

$$\frac{d}{dx}\begin{pmatrix} y_1-6 \\ y_2-3 \end{pmatrix} = \frac{d}{dx}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix}$$

$$\frac{d}{dx}\begin{pmatrix} y_1-4 \\ y_2-5 \end{pmatrix} = \frac{d}{dx}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix}$$

3. (5+2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2y_1 \\ 3y_1 + 4y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

Keresd meg A sajatertekeit es sajatvektorait!

$$A = \begin{pmatrix} 2 & 0 \\ 3 & 4 \end{pmatrix} \qquad 0 = \det(A - \lambda E) = \begin{vmatrix} 2-\lambda & 0 \\ 3 & 4-\lambda \end{vmatrix} = (2-\lambda)(4-\lambda) - 0 \cdot 3$$

$\lambda_1 = 2$

$(A - 2 \cdot E)\overline{v} = \overline{0}$

$$\begin{pmatrix} 2-2 & 0 \\ 3 & 4-2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$3x + 2y = 0, \quad y = -\frac{3}{2}x$

$$\overline{V}_1 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$\lambda_2 = 4$

$(A - 4 \cdot E)\overline{v} = \overline{0}$

$$\begin{pmatrix} 2-4 & 0 \\ 3 & 4-4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-2x = 0 = 3x \rightarrow x = 0$

$$\overline{V}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Ird fel a DE altalanos megoldasat!

$$\overline{y}(t) = C_1 e^{+\lambda_1 t}\overline{V}_1 + C_2 e^{+\lambda_2 t}\overline{V}_2 = C_1 \cdot e^{2t}\begin{pmatrix} 2 \\ -3 \end{pmatrix} + C_2 \cdot e^{+4t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Szamold ki a DE partikularis megoldasat!

$$\begin{pmatrix} 5 \\ 3 \end{pmatrix} = C_1 \begin{pmatrix} 2 \\ -3 \end{pmatrix} + C_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\left.\begin{aligned} 5 &= 2C_1 \\ 3 &= -3C_1 + C_2 \end{aligned}\right\} \rightarrow \begin{aligned} C_1 &= 5/2 \\ C_2 &= \frac{21}{2} \end{aligned}$$

$$\overline{Y}_{part}(t) = \frac{5}{2} e^{2t}\begin{pmatrix} 2 \\ -3 \end{pmatrix} + \frac{21}{2} e^{4t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

4. (2 + (1 + 2) + 5 pont)
a) $x_0 = 8$, $x_{n+1} = \phi(x_n) = x_n + 99$. Mennyi x_n ? X_n: számtani sorozat

$$X_n = 8 + n \cdot 99$$

Nincs fixpont: $X_f = X_f + 99 \longrightarrow$ nincs megoldás

b) Ird fel a kovetkezo Lagrange fuggvenyekhez tartozo Euler-Lagrange egyenlet(ek)et!

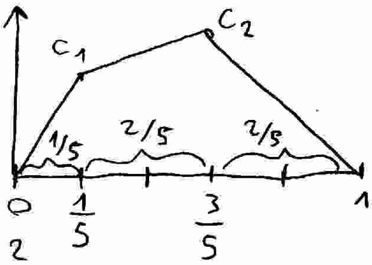
$$L = y - y^4, \qquad M = (y_1')(y_2')^2/2 + y_1'y_1 + y_2'y_1$$

$$L: \frac{d}{dx}\left(\frac{\partial L}{\partial y'}\right) - \frac{\partial L}{\partial y} = 0 = \frac{d}{dx} 0 - (1 - 4y^3) = 0 = 4y^3 - 1$$

$$M: \frac{d}{dx}\left(\frac{\partial L}{\partial y_1'}\right) - \frac{\partial L}{\partial y_1} = 0 = \frac{d}{dx}\left(\frac{y_2'^2}{2} + y_1\right) - (y_1' + y_2')$$

$$\frac{d}{dx}\left(\frac{\partial L}{\partial y_2'}\right) - \frac{\partial L}{\partial y_2} = 0 = \frac{d}{dx}\left(y_1' \cdot y_2' + y_1\right) - 0$$

c) Legyen adott az $S[u] = \int_0^1 (y'(x))^2 + (1 + x)(y(x))^2\,dx$ funkcional a $[0, 1]$-en ertelmezett es a vegpontokban eltuno fuggvenyek H teren. Legyen V a $[0, 1]$-en ertelmezett, a vegpontokban eltuno es a $[0, 1/5]$, $[1/5, 3/5]$, $[3/5, 1]$ intervallumokon affin folytonos fuggvenyek tere. Legyen ϕ_1 es ϕ_2 ennek e ternek egy bazisa, ahol $\phi_1(1/5) = \phi_2(3/5) = 1$ es $\phi_2(1/5) = \phi_1(3/5) = 0$. Legyen $u_h = c_1\phi_1 + c_2\phi_2$. Szamitsd ki az $S[u_h] = s(c_1, c_2)$ ketvaltozos fuggvenyt! (Az masodik tag kiszamitasara az integralban hasznalj valamilyen kozelito modszert es add is meg a modszer nevet!)



$(y'(x))^2$ $\qquad \Delta x$

$$S[U_h] = \left(\frac{c_1}{1/5}\right)^2 \cdot \frac{1}{5} + \left(\frac{c_2 - c_1}{2/5}\right)^2 \cdot \frac{2}{5} + \left(\frac{-c_2}{2/5}\right)^2 \cdot \frac{2}{5}$$

$$+ \frac{1}{2}\left((1+0)\cdot 0^2 + \left(1 + \frac{1}{5}\right)\cdot c_1^2\right)\cdot \frac{1}{5}$$

$$+ \frac{1}{2}\left(\left(1 + \frac{1}{5}\right)\cdot c_1^2 + \left(1 + \frac{3}{5}\right) c_2^2\right)\cdot \frac{2}{5}$$

$$+ \frac{1}{2}\left(\left(1 + \frac{3}{5}\right) c_2^2 + (1+1)\cdot 0^2\right)\cdot \frac{2}{5}$$

trapéz módszer