Diff. Egy. Vizsga. 17.05.19

1a) $y' = (y-1)(2-y)(y-3) = -y^3+6y^2-11y+6 = f(y)$

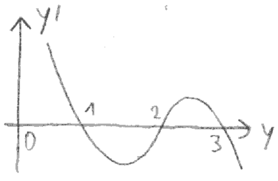
Fixpontok: $y_1 = 1,\ y_2 = 2,\ y_3 = 3$

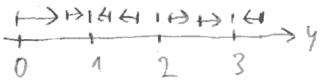
Lin. DE: $\dfrac{df(y)}{dy} = -3y^2+12y-11$

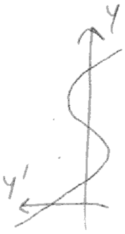
$\dfrac{d}{dt}(y-1) = \dfrac{d}{dt}\Delta y = f'(1)\Delta y = -2\Delta y$

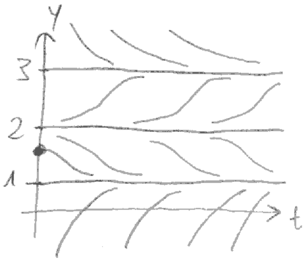
$\dfrac{d}{dt}(y-2) = \dfrac{d}{dt}\Delta y = f'(2)\Delta y = 1\,\Delta y$

$\dfrac{d}{dt}(y-3) = \dfrac{d}{dt}\Delta y = f'(3)\Delta y = -2\Delta y$









Ha $y(0) = 1.5$, $\lim\limits_{t\to\infty} y(t) = 1$, $\lim\limits_{t\to-\infty} = 2$

1b) $f(x) = \sum\limits_{n\in\mathbb{Z}} (e_n, f)\cdot \dfrac{e^{inx}}{\sqrt{2\pi}} = \sum\limits_{n\in\mathbb{Z}} \left(\int_0^{2\pi} \overline{\dfrac{e^{inx}}{\sqrt{2\pi}}}\cdot f(x)\,dx \right)\cdot \dfrac{e^{inx}}{\sqrt{2\pi}} =$

$\sum\limits_{n\in\mathbb{Z}} \left(\dfrac{1}{\sqrt{2\pi}} \int_1^2 4e^{-inx}\,dx \right)\cdot \dfrac{e^{inx}}{\sqrt{2\pi}} = \sum\limits_{n\in\mathbb{Z}} 4\cdot \dfrac{e^{-in\cdot 2} - e^{-in\cdot 1}}{-in}\cdot \dfrac{1}{2\pi}\cdot e^{inx}$

Mivel $3\partial_x^2 \left(\dfrac{e^{inx}}{\sqrt{2\pi}} \right) = -3n^2\cdot \dfrac{e^{inx}}{\sqrt{2\pi}}$, így

$\varphi(t,x) = \sum\limits_{n\in\mathbb{Z}} 4\cdot \dfrac{e^{-in\cdot 2} - e^{-in\cdot 1}}{-in}\cdot \dfrac{1}{2\pi}\cdot e^{inx}\cdot e^{-3n^2\cdot t}$

2a) $f(x) = (x-1)^{-1}$, $f'(x) = -(x-1)^{-2}$, $f''(x) = 2(x-1)^{-3}$, $f(2) = 1$, $f'(2) = -1$, $f''(2) = 2$

$f(2+\Delta x) = 1 - 1\cdot\Delta x + hiba(\Delta x)$, $|hiba(\Delta x)| \le \frac{1}{2}\Delta x^2\cdot \max\limits_{z\in[2,2+\Delta x]} |f''(z)| = \frac{1}{2}\Delta x^2\cdot 2$, ha $\Delta x \ge 0$

2b) $y' = (x-4)^2 + x$, $y(3) = 1$.

- Euler: $y(3.1) \approx 1 + [(3-1)^2+3]\cdot 0.1 = 1.7$
- Heun: $y(3.1) \approx 1 + \frac{1}{2}\left\{ [(3-1)^2+3] + [(3.1-1.7)^2+3.1] \right\}\cdot 0.1$

2c) $u''(x) \approx \dfrac{1}{\Delta x^2}\left(u(x+\Delta x) - 2u(x) + u(x-\Delta x) \right)$

$xu'' + x^2u' + u = 3$, $u(0) = u(1) = 0$, $\Delta x = 1/5$

$$\begin{bmatrix} 1\Delta x & & & \\ & 2\Delta x & & \\ & & 3\Delta x & \\ & & & 4\Delta x \end{bmatrix} \cdot \frac{1}{\Delta x^2} \begin{bmatrix} -2 & 1 & & \\ 1 & -2 & 1 & \\ & 1 & -2 & 1 \\ & & 1 & -2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + \begin{bmatrix} (1\Delta x)^2 & & & \\ & (2\Delta x)^2 & & \\ & & (3\Delta x)^2 & \\ & & & (4\Delta x)^2 \end{bmatrix} \cdot \frac{1}{\Delta x} \begin{bmatrix} -1 & 1 & & \\ & -1 & 1 & \\ & & -1 & 1 \\ & & & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} +$$

$$+ \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

③ $\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$. $\quad 0 = \begin{vmatrix} 2-\lambda & 3 \\ 0 & 3-\lambda \end{vmatrix} \Rightarrow \lambda_1 = 2,\ \lambda_2 = 3$

Sajátvektorok: $\begin{pmatrix} 2-2 & 3 \\ 0 & 3-2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow 3v = 0,\ \overline{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 2-3 & 3 \\ 0 & 3-3 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow -u+3v = 0,\ \overline{v}_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$\overline{y}_{ált}(t) = C_1 e^{2t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{3t}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$. $\quad \overline{y}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \Rightarrow \overline{y}_{part} = e^{3t}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

---

④ A) $y'' + 2y' + 5y = t - e^{-4t}$, $y(0) = 4$, $y'(0) = 5$

$$\left(s^2 Y(s) - 4s - 5\right) + 2\left(sY(s) - 4\right) + 5Y(s) = \frac{1}{s^2} - \frac{1}{s+4}$$

$$Y(s) = \frac{1}{s^2+2s+5}\cdot\left(\frac{1}{s^2} - \frac{1}{s+4} + 4s + 13\right)$$

$$= \frac{A}{s-(-1+2i)} + \frac{B}{s-(-1-2i)} + \frac{C}{s+4} + \frac{D}{s^2} + \frac{E}{s}$$

$$y(t) = A e^{(-1+2i)t} + B e^{(-1-2i)t} + C e^{-4t} + Dt + E$$

B) $y'' - 9y = f(t)$

1. $G''(t) - 9G(t) = \delta(t)$

a) G retardált: $G(t) = 0$, ha $t < 0$

b) $G(0^+) = 0$, $G'(0^+) = 1$, mivel ha $t \approx 0$, $G''(t) \approx \delta(t)$

c) $G''(t) - 9G(t) = 0$, ha $t > 0$, tehát

$G(t) = C_1 \cdot e^{3t} + C_2 e^{-3t}$

$$b) + c) \Rightarrow G(t) = \begin{cases} 0, & \text{ha } t < 0 \\ \frac{1}{6}\left(e^{3t} - e^{-3t}\right), & \text{ha } t > 0 \end{cases}$$

2. $$y(t) = (G*f)(t) = \int_{-\infty}^{\infty} G(t-\tau) f(\tau)\, d\tau = \int_{-\infty}^{t} \frac{1}{6}\left(e^{3(t-\tau)} - e^{-3(t-\tau)}\right) f(\tau)\, d\tau =$$

$$= \int_{0}^{t} \frac{1}{6}\left(e^{3(t-\tau)} - e^{-3(t-\tau)}\right) f(\tau)\, d\tau$$

mivel $f(t) = 0$, ha $t < 0$.

3. $$y(t) = 7\cdot G(t) + \int_{0}^{\infty} G(t-\tau) f(\tau)\, d\tau =$$

$$= 7\cdot\frac{1}{6}\left(e^{3t} - e^{-3t}\right) + \int_{0}^{t} \frac{1}{6}\left(e^{3(t-\tau)} - e^{-3(t-\tau)}\right) f(\tau)\, d\tau$$