Lev.Vizsga, Diff.Egy., 2017.05.13. NEPTUN: :

Név: Aláírás:

1. (2+3+2+3 pont)

a) Ird at a kovetkezo DE rendszert elsorendu idofuggetlen DE rendszerre!

$$\frac{d^2}{dt^2}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} ty_1' y_1^2 \\ t^2 y_2' + y_1' - \cos(t) \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ v_1 \\ y_2 \\ v_2 \\ s \end{pmatrix} = \begin{pmatrix} v_1 \\ s\, v_1 y_1^2 \\ v_2 \\ s^2 v_2 + v_1 - \cos(s) \\ 1 \end{pmatrix}$$

b) Alkalmazd az Euler, illetve a Heun modszert a kovetkezo DE-re $\Delta x = 0.1$ lepeskozzel az $y(3) = 1$ kezdeti feltetel mellett!

$$y' = -y^2 - x^2.$$

Mit josol a ket modszer $y(3.1)$-re?

Euler:

$$y(3.1) \simeq 1 + (-1^2 - 3^2)\cdot 0.1 = 0$$

Heun: $k_1 = -1^2 - 3^2 = -10,\quad k_2 = -0^2 - 3.1^2$

$$y(3.1) \approx 1 + \frac{1}{2}\left[(-10) + (-0^2 - 3.1^2)\right]\cdot 0.1$$

c) Ird fel a kovetkezo Lagrange fuggvenyhez tartozo Euler-Lagrange egyenletet!

$$L(x, u, u') = u'^2 + uu' + u^4,$$

$$\frac{d}{dx}\frac{\partial L}{\partial u'} - \frac{\partial L}{\partial u} = 0$$

$$0 = \frac{d}{dx}(2u' + u) - (u' + 4u^3) = 2u'' + u' - u' - 4u^3 = 2u'' - 4u^3 = 0$$

d) Legyen $y'(t) = (t+2)y + t - 1$; $y(2) = 3$. Ird fel $y(2 + \Delta t)$ masodrendu Taylor polinomjat!

$$y(2) = 3, \quad y'(2) = (2+2)\cdot 3 + 2 - 1 = 13$$

$$y''(t) = \left(\frac{\partial}{\partial t} + [(t+2)y + t - 1]\frac{\partial}{\partial y}\right)[(t+2)y + t - 1] =$$

$$= (y+1) + [(t+2)y + t - 1]\cdot(t+2), \qquad y''(2) = (3+1) + [(2+2)\cdot 3 + 2 - 1]\cdot(2+2) = 56$$

$$y(2+\Delta t) \approx 3 + 13\Delta t + \frac{1}{2}\cdot 56\,\Delta t^2$$

3a. (5 pont)

$y' = (y^3 - 27)y. \; = y^4 - 27y$ $\qquad \frac{d}{dy}(y^4 - 27y) = 4y^3 - 27$

Keresd meg a DE fixpontjait!

1p $\quad (y^3 - 27)y = 0 \quad \Rightarrow \quad y_1 = 0, \; y_2 = 3$

Ird fel a fixpontok koruli linearizalt kozelito DE-t!

1: $\quad \frac{d}{dx}(y-0) = \frac{d}{dx}\Delta y = (4 \cdot 0^3 - 27)\Delta y = -27\Delta y$

1p

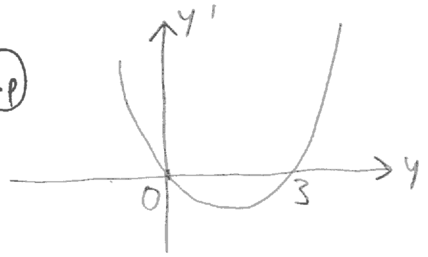
2: $\quad \frac{d}{dx}(y-3) = \frac{d}{dx}\Delta y = (4 \cdot 3^3 - 27)\Delta y = 81\Delta y$

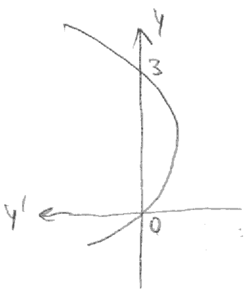
Ha $y(0) = 1.5$, mennyi

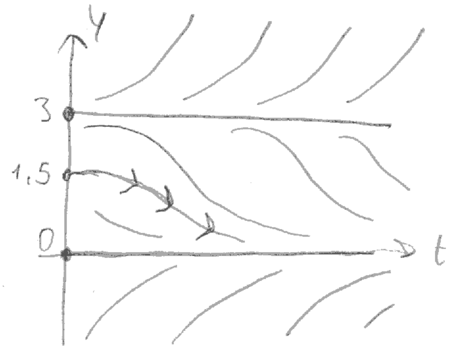
1p $\lim_{x\to\infty} y(x) = 0$ $\qquad \lim_{x\to-\infty} y(x) = 3$

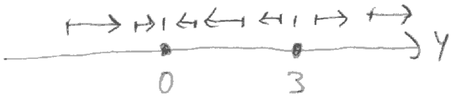
Vazold a DE megoldasgorbeit!

2p









3b. (5 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} (3y_2 + 6)y_1 \\ y_2(2y_1 + 4). \end{pmatrix}$$

Keresd meg a DE fixpontait!

$$\left.\begin{aligned} (3y_2+6)y_1 &= 0 \\ y_2(2y_1+4) &= 0 \end{aligned}\right\} \Rightarrow \begin{matrix} [y_1 = 0 \text{ és } y_2 = 0] \\ \text{vagy} \\ [y_2 = -2 \text{ és } y_1 = -2] \end{matrix} \qquad P_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \; P_2 = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

2p

$$\mathrm{Jac} = \begin{bmatrix} \partial_{y_1}[(3y_2+6)y_1] & \partial_{y_2}[(3y_2+6)y_1] \\ \partial_{y_1}[y_2(2y_1+4)] & \partial_{y_2}[y_2(2y_1+4)] \end{bmatrix} =$$

Ird fel a fixpont koruli linearizalt kozelito DE-t!

1p

$$\mathrm{Jac} = \begin{pmatrix} 3y_2+6 & 3y_1 \\ 2y_2 & 2y_1+4 \end{pmatrix}, \quad \mathrm{Jac}(P_1) = \begin{pmatrix} 6 & 0 \\ 0 & 4 \end{pmatrix}, \quad \mathrm{Jac}(P_2) = \begin{pmatrix} 0 & -6 \\ -4 & 0 \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} y_1 - 0 \\ y_2 - 0 \end{pmatrix} = \frac{d}{dt}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix}, \quad \frac{d}{dt}\begin{pmatrix} y_1 - (-2) \\ y_2 - (-2) \end{pmatrix} = \frac{d}{dt}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix} = \begin{pmatrix} 0 & -6 \\ -4 & 0 \end{pmatrix}\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix}$$

1p $\qquad\qquad$ 1p

2. (5+2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2y_1 + 3y_2 \\ 3y_1 + 2y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$$

Keresd meg A sajatertekeit es sajatvektorait!

$$0 = \det(A - \lambda E) = \begin{vmatrix} 2-\lambda & 3 \\ 3 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 - 3^2 = \lambda^2 - 4\lambda - 5 = 0 \Rightarrow \lambda_1 = 5, \; \lambda_2 = -1$$

$$\begin{pmatrix} 2-5 & 3 \\ 3 & 2-5 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -3 & 3 \\ 3 & -3 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Longrightarrow u = v, \quad \overline{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2-(-1) & 3 \\ 3 & 2-(-1) \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Longrightarrow u = -v, \quad \overline{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Ird fel a DE altalanos megoldasat!

$$\overline{y}_{ált}(t) = c_1 e^{5t} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-1 \cdot t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Szamold ki a DE partikularis megoldasat!

$$c_1 \cdot 1 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \cdot 1 \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\left.\begin{matrix} c_1 + c_2 = 3 \\ c_1 - c_2 = 1 \end{matrix}\right\} \Longrightarrow c_1 = 2, \quad c_2 = 1$$

$$\overline{y}_{part}(t) = 2 e^{5t} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 1 \cdot e^{-t} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

((2+2+1)+(3+1+1) pont)

A) $y'' - 5y' + 6y = (t+3)^2$, $y(0) = 5$, $y'(0) = 7$. Mennyi $Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$)

$$\underbrace{(t+3)^2}_{= t^2+3t+9}$$

$$\left(s^2 Y(s) - 5s - 7\right) - 5\left(sY(s) - 5\right) + 6Y(s) = \frac{2}{s^3} + 6\cdot\frac{1}{s^2} + \frac{9}{s}$$

$$Y(s) = \frac{1}{s^2 - 5s + 6}\cdot\left(\frac{2}{s^3} + 6\cdot\frac{1}{s^2} + \frac{9}{s} + 5s + \overbrace{7 - 25}^{-18}\right)$$

Ird fel $Y(s)$ parcialis tort felbontasat! (Az egyutthatokat nem kell kiszamolni.)

$$s^2 - 5s + 6 = 0 \Rightarrow s_1 = 2,\ s_2 = 3$$

$$Y(s) = \frac{A}{s-2} + \frac{B}{s-3} + \frac{C}{s^3} + \frac{D}{s^2} + \frac{E}{s}$$

Mennyi $y(t)$?

$$y(t) = A e^{2t} + B e^{3t} + \frac{C}{2} t^2 + Dt + E$$

B) Oldd meg a $y' + 9y = f(t)$ DE-t: 1.Keresd meg a G retardalt Green fuggvenyt!

$$\left.\begin{array}{l} G(t) = 0,\ \text{ha } t < 0 \\ G(0^+) = 1 \\ \underbrace{G'(t) + 9G(t) = 0}_{\Downarrow\ G(t) = C\cdot e^{-9t}},\ \text{ha } t > 0 \end{array}\right\} \Rightarrow G(t) = \begin{cases} 0, & \text{ha } t \\ e^{-9t}, & \text{ha } t < 0 \end{cases}$$

2. G segitsegevel fejezd ki y-t, ha $y(t) = f(t) = 0$ amikor $t \ll 0$!

$$y(t) = \int_{-\infty}^{\infty} G(t-\tau) f(\tau)\, d\tau = \int_{-\infty}^{t} e^{-9(t-\tau)} f(\tau)\, d\tau$$

3. Hasznald G-t arra, hogy kifejezd a megoldast $t > 0$-ra, ha $y(0) = 7$!

$$y(t) = 7\cdot e^{-9t} + \int_0^{\infty} G(t-\tau) f(\tau)\, d\tau =$$

$$= 7\cdot e^{-9t} + \int_0^{t} e^{-9(t-\tau)} f(\tau)\, d\tau$$