1. • Ird at a kovetkezo egyenletet elsorendu DE rendszerre!

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$$y''' = y + y';$$

• Oldd meg!

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$$y''(x) = x, \quad y(0) = 2,\ y'(0) = 3,$$

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$$y''(x) = 1, \quad y(0) = 2,\ y(2) = 3.$$

2. Alkalmazd az Euler, illetve a Heun modszert a kovetkezo DE-re $\Delta x = 0.1$ lepeskozzel az $y(3) = 2$ kezdeti feltetel mellett! 2 3

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$$y' = f(x, y) = y - x^2;$$

Mit josolnak ezek a modszerek $y(\cancel{2.1})$ 3.1-re?

3. $y' = -y^2 + y$.

Keresd meg a DE fixpontjait!

Ird fel a fixpontok koruli linearizalt kozelito DE-t!

6 Ha $y(0) = 0.3567$, mennyi

$\lim_{x\to\infty} y(x) =$ $\lim_{x\to-\infty} y(x) =$

Vazold a DE megoldasgorbeit!

4.

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_1 - 2 \\ (y_2 - 3)(y_1 - 4) \end{pmatrix}.$$

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2 Keresd meg a DE fixpontjat!

4 Ird fel a fixpont koruli linearizalt kozelito DE-t!

6 5. 3 (a) Oldd meg: $y'' + 4y = 0$, $y(0) = 1$, $y'(0) = 2$.

~~3 (b) Oldd meg az allando varialasanak a modszerevel: $y' - y = e^{-t} + 2$.~~

5 6. 3 (a) Oldd meg a $G' - G = \delta$ egyenletet, ha $G(t) = 0$ negativ t-kre!

2 (b) Ird fel G segitsegevel az $y' - y = f(t)$ egyenlet megoldasat, ha $y(t) = f(t) = 0$ negativ t-kre!

7. $y'' + 9y = 15$, $y(0) = 4$, $y'(0) = 2$

2 (a) Szamold ki $y(t)$-nek az $Y(s)$ Laplace transzformaltjat!

6 2 (b) Ird fel $Y(s)$ parcialis tort felbontasanak a strukturajat!

2 (c) Mennyi $y(t)$?

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① a) $y'=v,\ y''=a \quad \frac{d}{dt}\begin{pmatrix} y \\ v \\ a \end{pmatrix} = \begin{pmatrix} v \\ a \\ y+v \end{pmatrix} \iff y'''=y+y'$

b) $y''=x \longrightarrow y'=\int x\,dx = \frac{x^2}{2}+C_1 \longrightarrow y=\int \frac{x^2}{2}+C_1\,dx = \frac{x^3}{6}+C_1x+C_2$

$$\left.\begin{array}{l} y(0)=2 \longrightarrow \frac{0^3}{6}+C_1\cdot 0+C_2=6 \\ y'(0)=3 \longrightarrow \frac{0^2}{2}+C_1=3 \end{array}\right\} \longrightarrow y=\frac{x^3}{6}+3x+2$$

c) $y''=1 \longrightarrow y'=\int 1\,dx = x+C_1 \longrightarrow y=\int x+C_1\,dx = \frac{x^2}{2}+C_1x+C_2$

$$\left.\begin{array}{l} y(0)=2 \longrightarrow \frac{0^2}{2}+C_1\cdot 0+C_2=2 \\ y(2)=3 \longrightarrow \frac{2^2}{2}+C_1\cdot 2+C_2=3 \end{array}\right\} \longrightarrow y=\frac{x^2}{2}+2-\frac{1}{2}x$$

② Euler: $y(3)=2$, $y'=y-x^2$

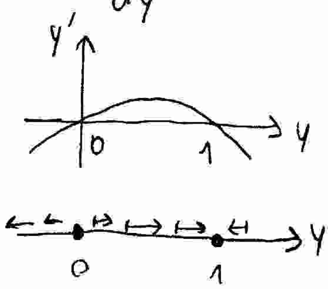
$$y(3.1) \approx 2+(2-3^2)\cdot 0.1 = 1.3$$

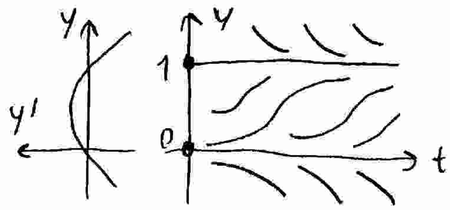
Heun: $y(3.1) \approx 2+\frac{1}{2}\left[(2-3^2)+(1.3-3.1^2)\right]\cdot 0.1$

③ $y'=-y^2+y=-y(y-1)$ Fixpontok: $-y(y-1)=0 \longrightarrow y_1=0,\ y_2=1$

$\frac{d(-y^2+y)}{dy}=-2y+1$. Lin.DE: $\frac{d}{dt}(y-0)=\frac{d}{dt}\Delta y_1=(-2\cdot 0+1)\Delta y_1=1\cdot\Delta y_1$

$\frac{d}{dt}(y-1)=\frac{d}{dt}\Delta y_2=(-2\cdot 1+1)\Delta y_2=-1\cdot\Delta y_1$





④ $\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_1-2 \\ (y_2-3)(y_1-4) \end{pmatrix}$ Fixpont: $\left.\begin{array}{l} y_1-2=0 \\ (y_2-3)(y_1-4)=0 \end{array}\right\} \longrightarrow \begin{array}{l} y_1=2 \\ y_2=3 \end{array} \quad P_1=\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$\text{JacobiMátrix} = \begin{pmatrix} \frac{\partial}{\partial y_1}(y_1-2) & \frac{\partial}{\partial y_2}(y_1-2) \\ \frac{\partial}{\partial y_1}[(y_2-3)(y_1-4)] & \frac{\partial}{\partial y_2}[(y_2-3)(y_1-4)] \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ y_2-3 & y_1-4 \end{pmatrix}$$

$\mathrm{Jac}(P_1)=\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix} \qquad \frac{d}{dt}\begin{pmatrix} y_1-2 \\ y_2-3 \end{pmatrix} = \frac{d}{dt}\overline{\Delta y} = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}\overline{\Delta y}$

5. a) $y''+4y=0$, $y=e^{\lambda t} \to \lambda^2+4=0 \to \lambda_{1,2}=\pm 2i$

$y=C_1\cos(2t)+C_2\sin(2t)$ vagy $\tilde{C}_1 e^{2it}+\tilde{C}_2 e^{-2it}$

$y'=-2C_1\sin(2t)+2C_2\cos(2t)$

$$\left.\begin{array}{l} y(0)=1 \to C_1\cdot 1+C_2\cdot 0=1 \\ y'(0)=2 \to -2C_1\cdot 0+2C_2\cdot 1=2 \end{array}\right\} \to y=1\cdot\cos(2t)+1\cdot\sin(2t)$$

b) $y'-y=e^{-t}+2$. $y'-y=0 \to y=c\cdot e^t$

$y=C(t)\cdot e^t$

$y'=C'e^t+Ce^t$

$(C'e^t+Ce^t)-Ce^t=e^{-t}+2 \to c'e^t=e^{-t}+2 \to C'=e^{-2t}+2e^{-t}$

$\to C=\int e^{-2t}+2e^{-t}\,dt=\frac{e^{-2t}}{-2}+2\cdot\frac{e^{-t}}{-1}+k$

$\to y=C\cdot e^t=-\frac{1}{2}e^{-t}-2+k\cdot e^t$

6. a) $G'-G=\delta$

$t<0$: $G(t)=0$

$$\left.\begin{array}{l} t\approx 0:\ G'(t)=\delta(t) \to G(0^+)-G(0^-)=1 \\ t>0:\ G'-G=0 \to G(t)=e^t \end{array}\right\} \to G(t)=\begin{cases} 0, & \text{ha } t<0 \\ e^t, & \text{ha } t>0 \end{cases}$$

b) $y(t)=\int_{-\infty}^{\infty} G(t-\tau)f(\tau)\,d\tau=\int_{-\infty}^{t} e^{t-\tau}f(\tau)\,d\tau=\int_0^t e^{t-\tau}f(\tau)\,d\tau$

7. a) $y''+9y=15$, $y(0)=4$, $y'(0)=2$

$(s^2Y(s)-4s-2)+9Y(s)=\frac{15}{s}$

$Y(s)=\frac{1}{s^2+9}\left(4s+2+\frac{15}{s}\right)$

b) $Y(s)=\frac{A}{s+3i}+\frac{B}{s-3i}+\frac{C}{s}$

c) $y(t)=Ae^{-3it}+Be^{3it}+C$