Anal. III. Zh2.

1. (a) Oldd meg: $y' - 5y = 0,\ y(0) = 2$.
   (b) Oldd meg az allando varialasanak a modszerevel: $y' - 5y = e^{3t}$.
2. (a) Oldd meg a $G' - 5G = \delta$ egyenletet, ha $G(t) = 0$ negativ t-kre!
   (b) Ird fel G segitsegevel az $y' - 5y = f(t)$ egyenlet megoldasat, ha $y(t) = f(t) = 0$ negativ t-kre!
3. $y' - 5y = 3,\ y(0) = 2$
   (a) Szamold ki $y(t)$-nek az $Y(s)$ Laplace transzformaltjat!
   (b) Szamold ki $Y(s)$ parcialis tort felbontasat!
   (c) Mennyi $y(t)$?
4. (a) Szamitsd ki a Laplace tr. definicioja alapjan: $\mathcal{L}(e^{5t-7})$
   (b) Szamitsd ki a Laplace tr. definicioja alapjan: $\mathcal{L}(H(t-4)e^{5t})$
   (c)
   - Szamold ki az $f(t) = e^{-4t}$ es a $g(t) = e^{5t}$ fuggvenyek $h = f * g$ konvoluciojat!
   - Mennyi $\mathcal{L}(f(t))\mathcal{L}(g(t)) - \mathcal{L}(h(t))$?
5. (a) Oldd meg! $y'' - 100y = 0,\ y(0) = 0,\ y'(0) = 1$.
   (b) Oldd meg! $G'' - 100G = \delta$, es $G(t) = 0$ negativ t-kre.
   (c) Ird fel G segitsegevel az $y'' - 100y = f(t)$ egyenlet megoldasat, ha $y(t) = f(t) = 0$ negativ t-kre!
6. (a) Legyen $f_1 = (1/\sqrt{2}, i/\sqrt{2})^T$, $f_2 = (i/\sqrt{2}, z)^T$ egy ortonormalt bazis. Mennyi z ?
   (b) A $v = (4,2)^T$ vektor kifejezheto az f-ek linearis $\alpha f_1 + \beta f_2$ kombinaciojakent! Mennyi α ?
7. (a) Legyen $f(x) = H(-t) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$, ha $x \in (-\pi, \pi)$ Mennyi $\hat{f}_3$?
   (b) Legyen

$$\phi(0,x) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}, \qquad \phi(t,x) = \sum_{n\in\mathbb{Z}} d_n(t) \frac{e^{inx}}{\sqrt{2\pi}}, \qquad \partial_t \phi(t,x) = 5\partial^2_{xx}\phi(t,x).$$

   - Ird fel a $d_n(t)$ fuggvenyekre vonatkozo kozonseges DE-ket (kezdeti feltetellel egyutt)!
   - Mennyi $d_3(t)$?

Anal. III. zh2.

(1a) $y = 2 \cdot e^{5t}$

(1b) $y = C \cdot e^{5t}$, $y' = C' e^{5t} + C \cdot 5 e^{5t}$, $C' e^{5t} = e^{3t}$, $C' = e^{-2t}$

$C = \frac{e^{-2t}}{-2} + k$, $y = -\frac{1}{2} e^{3t} + k \cdot e^{5t}$

(2a) $t < 0$: $G(t) = 0$, $t \approx 0$: $G' \approx \delta$, $G(0^+) - G(0^-) = 1$

$t > 0$: $G(0^+) = 1$, $G' - 5G = 0 \longrightarrow G(t) = e^{5t}$

$G(t) = \begin{cases} 0, & \text{ha } t<0 \\ e^{5t}, & \text{ha } t>0 \end{cases}$

(2b) $y(t) = \int_{-\infty}^{\infty} G(t-\tau) f(t) d\tau = \int_{-\infty}^{t} e^{5(t-\tau)} f(\tau) d\tau$

(3a) $sY(s) - 2 - 5Y(s) = \frac{3}{s}$, $Y(s) = \frac{1}{s-5}\left(2 + \frac{3}{s}\right)$

(3b) $Y(s) = \frac{A}{s-5} + \frac{B}{s} \longrightarrow As + B(s-5) = 2s + 3 \longrightarrow A = 3\frac{2}{3}$, $B = -\frac{5}{3}$

(3c) $y(t) = 3\frac{2}{3} e^{5t} - \frac{5}{3}$

(4a) $\int_0^{\infty} e^{-st} e^{5t-7} dt = \int_0^{\infty} e^{-7} \cdot e^{-(s-5)t} dt = e^{-7}\left[\frac{e^{-(s-5)t}}{-(s-5)}\right]_0^{\infty} = \frac{e^{-7}}{s-5}$

(4b) $\int_0^{\infty} e^{-st} H(t-4) e^{5t} dt = \int_4^{\infty} e^{-(s-5)t} dt = \left[\frac{e^{-(s-5)t}}{-(s-5)}\right]_4^{\infty} = \frac{e^{-(s-5)\cdot 4}}{s-5}$

(4c) $h(t) = \int_0^t e^{-4(t-\tau)} e^{5\tau} d\tau = e^{-4t} \int_0^t e^{9\tau} d\tau = e^{-4t}\left(\frac{e^{9t}}{9} - \frac{1}{9}\right)$

$\mathcal{L}(f(t)) \mathcal{L}(g(t)) = \mathcal{L}((f * g)(t)) \longrightarrow$ válasz $= 0$

(5a) $\lambda^2 - 100\lambda = 0$, $\lambda = \pm 10$, $y = C_1 e^{10t} + C_2 e^{-10t}$

$y' = 10C_1 e^{10t} - 10C_2 e^{-10t}$

$y(0) = 0 \longrightarrow C_1 + C_2 = 0$

$y'(0) = 1 \longrightarrow 10C_1 - 10C_2 = 1$

$y(t) = \frac{1}{20}(e^{10t} + e^{-10t})$

(5b) $t < 0$, $G(t) = 0$

$t \approx 0$, $G''(t) = \delta \longrightarrow G(0^+) - G(0^-) = 0$, $G'(0^+) - G'(0^-) = 1$

$t > 0$, $G(0^+) = 0$, $G'(0^+) = 1$, $G'' - 100G = 0$ tehát $G(t) = \begin{cases} 0, & t<0 \\ \frac{1}{20}(e^{10t} - e^{-10t}), & t>0 \end{cases}$

(5c) $y(t) = \int_{-\infty}^{\infty} G(t-\tau) f(\tau) d\tau = \int_{-\infty}^{t} \frac{1}{20}\left(e^{+10(t-\tau)} - e^{-10(t-\tau)}\right) f(\tau) d\tau$

6a $(f_1, f_2) = \overline{\frac{1}{\sqrt{2}}} \cdot \frac{i}{\sqrt{2}} + \overline{\frac{i}{\sqrt{2}}} \cdot z = \frac{i}{2} - \frac{iz}{\sqrt{2}} = 0 \longrightarrow z = \frac{1}{\sqrt{2}}$

6b $\alpha = (f_1, v) \doteq \overline{\frac{1}{\sqrt{2}}} \cdot 4 + \overline{\frac{i}{\sqrt{2}}} \cdot 2 = \frac{1}{\sqrt{2}}(4 - 2i) = 2\sqrt{2} - \sqrt{2}\, i$

7a $\hat{f}_3 = \left(\frac{e^{i3x}}{\sqrt{2\pi}}, H(-x) \right) = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \overline{e^{i3x}} \cdot H(-x)\, dx \qquad e^{-i3(-\pi)} = e^{i\pi} = -1$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{0} e^{-i3x} dx = \frac{1}{\sqrt{2\pi}} \left[\frac{e^{-i3x}}{-i \cdot 3} \right]_{-\pi}^{0} = \frac{1}{\sqrt{2\pi}} \left(\frac{1}{-i \cdot 3} - \frac{-1}{-i \cdot 3} \right)$$

$$= i \sqrt{\frac{2}{\pi}} \cdot \frac{1}{3}$$

7b $d_n'(t) = 5 \cdot (-n^2)\, d_n(t), \quad d_n(0) = \hat{f}_n$

$d_3(t) = \frac{i}{3} \sqrt{\frac{2}{\pi}} \cdot e^{-5 \cdot 3^2 \cdot t}$