Differential equations. Quiz 2. **Name:** **Neptun:**

1. Let $e_1 = \left(\frac{\sqrt{3}i}{2}, \frac{1}{2}\right)^T$, $e_2 = \left(\frac{1}{2}, \frac{\sqrt{3}i}{2}\right)^T$ be an othonormed basis in $\mathbb{C}^2$ and let $u = (3+i, 4-2i)^T$, $u = \alpha_1 e_1 + \alpha_2 e_2$. Compute α_2 !
2. The characteristic function χ_D is defined as $\chi_D(x) = 0$ if $x \notin D$, otherwise $\chi_D(x) = 1$. Let $f(x) = \chi_{[-2,2]}(x)$, $f(x) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$. Compute $\hat{f}_9$!
3. $\partial_t \phi(t,x) = \partial_{xx}\phi(t,x)$, $\phi(0,x) = \cos(3\pi x) + 4\cos(5\pi x)$. Compute $\phi(t,x)$!

(1) $\alpha_2 = (e_2, u) = (e_2, \alpha_1 e_1 + \alpha_2 e_2) = \alpha_1 \overbrace{(e_2, e_1)}^{0} + \alpha_2 \overbrace{(e_2, e_2)}^{1}$ (2)

$$\alpha_2 = \left(\begin{pmatrix} 1/2 \\ \frac{\sqrt{3}i}{2} \end{pmatrix}, \begin{pmatrix} 3+i \\ 4-2i \end{pmatrix} \right) = \overline{\frac{1}{2}} \cdot (3+i) + \overline{\frac{\sqrt{3}i}{2}} \cdot (4-2i) = \frac{1}{2}(3+i) - \frac{\sqrt{3}\cdot i}{2}(4-2i)$$

$=$ (2)

(2) $\hat{f}_9 = (e_9, f) = \int_{-\pi}^{\pi} \overline{\frac{e^{i\cdot 9\cdot x}}{\sqrt{2\pi}}} f(x)\,dx = \frac{1}{\sqrt{2\pi}} \int_{-2}^{2} e^{-9ix}\,dx = \frac{1}{\sqrt{2\pi}} \left[\frac{e^{-9ix}}{-9i} \right]_{-2}^{2}$ (2)

$$= \frac{1}{\sqrt{2\pi}} \frac{e^{-18i} - e^{18i}}{-4.5 \cdot 2i} = \frac{1}{\sqrt{2\pi}\cdot 4.5} \cdot \sin(18)$$

(3) $\partial_{xx}[\cos(3\pi x)] = -(3\pi)^2 \cos(3\pi x)$

$\partial_{xx}[\cos(5\pi x)] = -(5\pi)^2 \cos(5\pi x)$,

so

$$\varphi(t,x) = e^{-9\pi^2 t} \cdot \cos(3\pi x) + 4 \cdot e^{-25\pi^2 t} \cdot \cos(5\pi x)$$

(5)

Max: 13 points