Differential equations. quiz.2.problems

1. Let $u = (1, i)^T$, $v = (3 - i, 2 + 4i)^T$. Compute (u, v) and (v, u) !
2. $e_1 = \frac{1}{\sqrt{2}}(1, i)^T$, $e_2 = \frac{1}{\sqrt{2}}(i, 1)^T$, $u = (3 + i, 4 - 2i)^T$, $u = \alpha_1 e_1 + \alpha_2 e_2$. Compute $\alpha_{1,2}$!
3. If $e_1 = \frac{1}{\sqrt{2}}(i, i)^T$, $e_2 = \frac{1}{\sqrt{2}}(i, \alpha)^T$ is an orthonormed basis, then how much is α?
4. Let
$$A = \begin{pmatrix} i & 3 - 2i \\ 4 + 2i & 8 - i \end{pmatrix}.$$
Compute A^*!
5.
$$B = \begin{pmatrix} 3 & 3 - 2i \\ 3 + 2i & 8 \end{pmatrix}.$$
If $\lambda_{1,2}$ are the eigenvalues of B, than how much is $Im(\lambda_1 - \lambda_2)$?
6.
$$B = \begin{pmatrix} 3i & 3i - 2 \\ 3i + 2 & 8i \end{pmatrix}.$$
If $\lambda_{1,2}$ are the eigenvalues of B, than how much is $Re(\lambda_1 - \lambda_2)$?
7. We claim that the set of functions $\sin(nx)$, $n = 1, 2, \ldots$ is an orthogonal complete basis in $\mathcal{H} = L^2([0, \pi], dx)$. Construct an orthonormed basis in $\mathcal{H}$!
8. Let f be a function on $[0, \pi]$ and express $f(x)$ as $\sum_{n>0}^{\infty} \hat{f}_n \sin(nx)$. Compute $\hat{f}_n$! Repeat this exercise for different orthonormed basis functions (ex. 9-11)!
9. We claim that the set of functions $\cos(nx)$, $n = 0, 2, \ldots$ is an orthogonal complete basis in $\mathcal{H} = L^2([0, \pi], dx)$. Construct an orthonormed basis in $\mathcal{H}$!
10. We claim that the set of functions $\sin(nx)$, $n = 1, 2, \ldots$ is an orthogonal complete basis in $\mathcal{H} = L^2([0, \pi], dx)$. Construct an orthonormed basis in $L^2([0, 88], dx)$!
11. We claim that the set of functions $\sin(nx)$, $n = 1, 2, \ldots$ is an orthogonal complete basis in $\mathcal{H} = L^2([0, \pi], dx)$. Construct an orthonormed basis in $L^2([-77, 88], dx)$!
12. In the sequel the characteristic function χ_D is defined as $\chi_D(x) = 0$ if $x \notin D$, otherwise $\chi_D(x) = 1$. Let $f(x) = \chi_{[0.1]}(x)$, $f(x) = \sum_{n \in \mathbb{Z}} \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$. Compute $\hat{f}_5$!
13. Let $f(x) = \chi_{[0.1]}(x) \in L^2(\mathbb{R}, dx)$. Compute $\hat{f}(2.3)$!
14. If $\phi(x, t)$ satisfies $(\partial_{tt}^2 - 9\partial_{xx}^2)\phi = 0$ and $\phi(x, t) = e^{i(kx + \omega t)}$, then what is the relation between k and ω ?
15. If $\phi(x, t)$ satisfies $(\partial_{tt}^2 - 9\partial_{xx}^2 - \partial_{xt}^2)\phi = 0$ and $\phi(x, t) = e^{i(kx + \omega t)}$, then what is the relation between k and ω ?
16. $\partial_t \phi(t, x) = \partial_{xx} \phi(t, x)$, $\phi(0, x) = \sin(4x) + 5\cos(6x)$. Compute $\phi(t, x)$!