

4a. (1+1+1+2 pont)

$$y' = (-y^2 + 4).$$

Find the fixed points of the DE!

If $y(0) = 0$, compute

$$\lim_{x \rightarrow \infty} y(x) = ? \quad \text{and} \quad \lim_{x \rightarrow -\infty} y(x) = ?$$

Sketch the $y(x)$ solution curves of the DE!

4b. (2+3 pont)

$$\frac{d}{dt} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -y_1 + 2 \\ 2y_2(-y_1 - 3y_2) \end{pmatrix}$$

Find the fixed points of the DE!

Write down the linearized DE around the fixed points!

1. (4+(3+3) pont)

a)

$$\frac{d}{dx} y = f(x, y) = x^2 + y - 2;$$

How much is y'' ? Write down the second order Taylor polynomial of $y(x)$ around $x = 0$, if $y(0) = 3$!

b) Apply the Euler and the Heun methods for the following DEs with stepsize $\Delta x = 0.01$!

$$\frac{d}{dt} y = y^2 + t^2, \quad y(2) = 3.$$

What are the predictions for $y(2.01)$?

Euler:

Heun:

2. (4+3+3 pont)

$$\frac{d}{dt} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -y_1 + 2y_2 \\ 2y_1 - y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Find the eigenvalues and eigenvectors of A !

Write down the general solution of the DE!

Compute the particular solution!

((3+2)+2+3 pont)

3a)

$$A = \begin{pmatrix} -1 & 0 \\ 6 & -1 \end{pmatrix}$$

How much is e^{tA} ?

Express the solution of the following DE

$$\frac{d}{dt} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -y_1 \\ 6y_1 - y_2 \end{pmatrix}, \quad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

with e^{tA} !

3b) Express the following DE as a first order system!

$$\frac{d^2}{dt^2} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_1^2 - y_2 \\ 2y_2 - 3y_1 \end{pmatrix}$$

3c)

Let $x_{n+1} = 1.2x_n - 20$, $x_0 = 1234$. How much is x_n ?