1. (2+2+3+3 pont)
A) Let $(\partial_x + 2\partial_t)(2\partial_x - \partial_t)\,\phi(x,t) = 0$.
1. Find the travelling wave solutions of the DE!
2. What is the speed of the forward and backward moving solutions?

B) Let

$$\partial_t \phi(t,x) = \partial_x^2 \phi(t,x), \quad \phi(t, x+2\pi) = \phi(t,x), \quad \phi(0,x) = f(x),$$

where $f(x) = 2019 \cdot sgn(x)$ on the interval $(-\pi, \pi)$.
1. Provide an orthonormed basis of $L^2(-\pi, \pi], dx)$!

2.Express f as a linear combination of that basis!

3. Express $\phi(t,x)$ using the result of the previous sub-exercise!

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2. (4+4+2 pont)
Use the definition of the Laplace-tr. for the computation of
a) $F(s) = \mathcal{L}(f(t)) = \mathcal{L}(e^{-t} - 3))$.
$F(s)=$

For what values of s do we have a well defined improper integral?

b) Compute the convolution of $f(t) = 1984$ and $g(t) = t$!

What is $\mathcal{L}(f(t))\mathcal{L}(g(t)) - \mathcal{L}(h(t))$?

c) $a_{n+1} = a_n - 112$, $a_0 = 78$. What is a_n ?

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2. ((1+3)+(3+3) pont)
A)

$$u''(x) + u'(x) + xu(x) = 93, \quad u(0) = u(1) = 0.$$

Approximate u by $\vec{u}_i = u(i\Delta x),\ i = 1, \ldots, 3,\ \Delta x = 1/4$.

- Give a reasonable approximation of $u''(x)$ using $u(x \pm \Delta x), u(x)$!
- What would be a resonable approximation of the DE for $\vec{u}$?

B)
Divide the $[0,1]$ by the following points: $x_i = 0.2,\ 0.4,\ 0.8$. Let $v(x)$ be a continuous function with the following values at $x = 0,\ 0.2,\ 0.4,\ 0.8,\ 1 \ : 0, v_1, v_2, v_3, 0$.

- Compute

$$Energy[v] = \int_0^1 (v')^2 + v' - (1-x)v\, dx$$

- Write down the Euler-Lagrange equations for $Energy[u]$!

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3. ((2+2+1)+2+3 pont)
A) $y'' - 2y' + 2y = 5(t-4)^2,\ y(0) = 2,\ y'(0) = 3$. What is $Y(s)$? $(\mathcal{L}(t^n) = \frac{n!}{s^{n+1}})$

Write down the partial fraction decomposition of $Y(s)$!

What is $y(t)$?

B) Let $f(x) = 1/x^3$. What is the linear approximation of f around $x_0 = 1$?
Estimate $|f(1+\Delta x) - f(1) - f'(1)\Delta x|$, if $\Delta x \in [0, 0.1]$!

C) Let $y'(t) = (y(t)(t+3),\ y(1) = 2$. What is the second order Taylor approximation of $y(1+\Delta t)$?!