

1a. (5 point)

$$y' = y(4 - y).$$

Find the fixed points of the DE!

Write down the linearized DE around the fixed points!

If $y(0) = 1$, what is

$$\lim_{x \rightarrow \infty} y(x) = \lim_{x \rightarrow -\infty} y(x) =$$

Plot the solution curves of the DE!

1b. (5 point)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} y_2 + 5 \\ (4 - y_1)(5 - y_2) \end{pmatrix}$$

Find the fixed points of the DE!

Write down the linearized DE around the fixed points!

2. (2+3+3+2 point)

2a. Let

$$\begin{pmatrix} 3 & 4 & 0 \\ 5 & 6 & 1 \\ 0 & 1 & 2 \end{pmatrix} = R + S + \lambda E,$$

where R is antisymmetric, S symmetric with zero trace and E is the unit matrix. What is R, S, λ ?

2b. Let

$$P \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} b \\ c \\ d \\ a \end{pmatrix}.$$

Provide an eigenvalue-eigenvector pair of P !

2c1. What is the retarded fundamental solution of $y''(t) - 2y'(t) + 2y(t) = \delta(t)$?

2c2. How much is the solution of the DE $y''(t) - 2y'(t) + 2y(t) = f(t)$, $y(t) = f(t) = 0$, ha $t \ll 0$?

3. (3+2+2+2+1 pont) Let

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2y_1 + 3y_2 \\ 3y_1 + 2y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$

3a. Find the eigenvalues and eigenvectors of A !

3b. Write down the general solution of the DE!

3c. Let

$$\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 13 \\ 77 \end{pmatrix}$$

What is the particular solution?

3d. How much is e^{tA} ?

3e. Express the solution of $\frac{d}{dt}\bar{y}(t) = A\bar{y}(t) + \bar{f}(t)$, $\bar{y}(t) = \bar{f}(t) = 0$, ha $t \ll 0$, with e^{tA} ?

4. (6+4 pont)

4a. Let

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -y_1 \\ 2y_1 - y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

How much is e^{tA} ?

If $(y_1(0), y_2(0))^T = (4, 5)$, what is the solution of the previous DE?

4b. Let

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -y_1 \\ -y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

How much is e^{tA} ?

If $(y_1(0), y_2(0))^T = (4, 5)$, what is the solution of the previous DE?