Név, Neptun. aláírás:

1.a. Legyen $y'(t) = \sin(3t^4)$, $y(5) = -5$. Fejezd ki $y(t)$-t hatarozott integralas segitsegevel!

$$y(t) = -5 + \int_5^t \sin(3t^4)\, dt$$

1.b. Legyen

$$\ddot{y} = \dot{y}^3 y^4 - t.$$

Irj fel egy elsorendu DE-t amelyik ekvivalens ezzel az egyenlettel!

$y_1 = y, \quad y_2 = \dot{y}$

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_2 \\ y_2^3 y_1^4 - t \end{bmatrix}$$

1.c. Legyen

$$\frac{d}{dt}y = (y-1)(t^2+1), \quad y(3) = 5.$$

Ird fel $y(3 + \Delta t)$ másodrendű ~~harmadrendu~~ Taylor polinomjat Δt szerint!

$$\ddot{y} = \frac{d}{dt}\dot{y} = \frac{d}{dt}\left[(y(t)-1)(t^2+1)\right] = \frac{\partial}{\partial t}\left([y-1)(t^2+1]\right) + \left[(y-1)(t^2+1)\right]\frac{\partial}{\partial y}\left[(y-1)(t^2+1)\right] =$$

$$= (y-1)\cdot 2t + \left[(y-1)(t^2+1)\right]\left[(t^2+1)\right]$$

$$\dot{y}(3,5) = (5-1)(3^2+1) = 40$$

$$\ddot{y}(3,5) = (5-1)\cdot 2\cdot 3 + \left[(5-1)(3^2+1)\right]\cdot\left[(3^2+1)\right] = 424$$

$$y(3+\Delta t) \approx 5 + 40\cdot\Delta t + \frac{424}{2!}\Delta t^2$$

1.d. Legyen $x_{n+1} = 4 - 2x_n$, $x_1 = 344$. Mennyi x_n ?

Fixpont: $x_{fix} = 4 - 2x_{fix} \longrightarrow x_{fix} = \frac{4}{3}$

$$x_n = (-2)^{n-1}\left(344 - \frac{4}{3}\right) + \frac{4}{3}$$

2. Legyen

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 7y_1 - 3y_2 \\ -3y_1 + 7y_2 \end{pmatrix} = A\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{illetve} \quad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Keresd meg A sajatertekeit es sajatvektorait!

Sajátérték: $\begin{vmatrix} 7-\lambda & -3 \\ -3 & 7-\lambda \end{vmatrix} = 0 = (7-\lambda)^2 - (-3)^2 = \lambda^2 - 14\lambda + 40 = 0 \longrightarrow \lambda_1 = 10 = (7+3),\ \lambda_2 = 4 = (7-3)$

$$\begin{bmatrix} 7-10 & -3 \\ -3 & 7-10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 7-4 & -3 \\ -3 & 7-4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\longrightarrow -3x - 3y = 0 \longrightarrow x = -y \qquad \longrightarrow 3x - 3y = 0 \longrightarrow x = y$$

$$\lambda_1 = 10 \quad \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \lambda_2 = 4 \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Ird fel a DE altalanos megoldasat!

$$\vec{y}_{\text{ált}}(t) = c_1 e^{10\cdot t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{4t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Szamold ki a DE partikularis megoldasait mindket kezdeti feltetel mellett!

$\vec{y}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$c_1\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\left.\begin{matrix} c_1 + c_2 = 1 \\ c_1 - c_2 = 0 \end{matrix}\right\} \longrightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$\vec{y}^{\,\mathrm{I}}_{\text{part}} = \frac{1}{2}e^{10\cdot t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{1}{2}e^{4t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} e^{10t} + e^{4t} \\ -e^{10t} + e^{4t} \end{bmatrix}$$

$\vec{y}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$c_1\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left.\begin{matrix} c_1 + c_2 = 0 \\ -c_1 + c_2 = 1 \end{matrix}\right\} \longrightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix}$$

$$\vec{y}^{\,\mathrm{II}}_{\text{part}} = -\frac{1}{2}e^{10t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{1}{2}e^{4t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -e^{10t} + e^{4t} \\ e^{10t} + e^{4t} \end{bmatrix}$$

Mennyi e^{tA} ?

$$e^{tA} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e^{\overset{\lambda_1}{10}\cdot t} & 0 \\ 0 & e^{\overset{\lambda_2 \to}{4}\cdot t} \end{bmatrix}\begin{array}{c} \begin{matrix} \vec{v}_1 & \vec{v}_2 \end{matrix} \\ \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}^{-1} \end{array} =$$

$$= \frac{1}{2}\begin{bmatrix} e^{10t} + e^{4t} & -e^{10t} + e^{4t} \\ \underbrace{-e^{10t} + e^{4t}}_{\vec{y}^{\,\mathrm{I}}_{\text{part}}} & \underbrace{e^{10t} + e^{4t}}_{\vec{y}^{\,\mathrm{II}}_{\text{part}}} \end{bmatrix}$$

3.a. Legyen

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 3y_2 - y_1 \\ -y_2 \end{pmatrix} = A\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Mennyi e^{tA} ?

$$\exp\left(t\begin{bmatrix} -1 & 3 \\ 0 & -1 \end{bmatrix}\right) = \exp\left(t\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\right) \cdot \exp\left(t\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}\right) =$$

$$= e^{-1 \cdot t}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + t \cdot \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}\right) = e^{-t}\begin{bmatrix} 1 & 3t \\ 0 & 1 \end{bmatrix}$$

Mi az elozo DE partikularis megoldasa az $(y_1(0), y_2(0))^T = (4, 5)$ kezdeti feltetel mellett?

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = e^{-t}\begin{bmatrix} 1 & 3t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} = e^{-t}\begin{bmatrix} 4+5 \cdot 3 \cdot t \\ 5 \end{bmatrix}$$

3.b. Legyen $f(x) = x^3$, $x_0 = 3$. Ird fel $f(3 + \Delta x)$ linearis approximaciojat, illletve adj egy felso becslest ennek a hibajara, ha $\Delta x = 1/10$!

$f(x) = x^3$, $f'(x) = 3x^2$, $f''(x) = 6x$

$f(x_0) = 27$, $f'(x_0) = 3 \cdot 3^2 = 27$, $f''(x_0) = 6 \cdot 3 = 18$, $f''(3 + \frac{1}{10}) = 6 \cdot (3 + \frac{1}{10}) = 18.6$

Lin. Approx: $f(3+0.1) \approx 27 + 27 \cdot 0.1$

$$\text{hiba}(0.1) = \left| (3+0.1)^3 - [27 + 27 \cdot 0.1] \right| \leq$$

$$\leq \frac{1}{2}(0.1)^2 \cdot 18.6$$

$\max_{z \in [3, 3.1]} f''(z)$

3.c. Legyen

$$\frac{d}{dt}y = y^2(t^2 + 1), \quad y(3) = 2.$$

Mit josol Heun modszere $y(3.001)$ ertekere?

Euler: $y(3.001) \approx 2 + (2^2[3^2+1]) \cdot 0.001 = 2.040$

Heun: $y(3.001) \approx$

$$\approx 2 + \frac{1}{2}\left[2^2[3^2+1] + 2.040^2[3.001^2+1]\right] \cdot 0.001$$

4.a.
$\frac{dy}{dt} = y(4-3y)$.
Keresd meg a DE fixpontjait!

$$y_1 = 0, \qquad y_2 = \frac{4}{3}$$

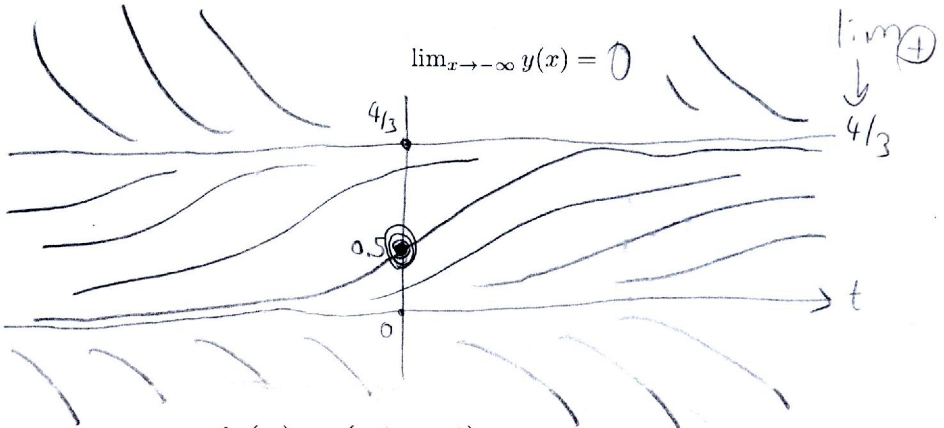
Ird fel a fixpontok koruli linearizalt kozelito DE-t!

$$\frac{d}{dy}\left(y(4-3y)\right) = 4-6y^2 \;\Big|\; \frac{d}{dt}(y-0) = \frac{d}{dt}\Delta y = \overbrace{\left(4-6\cdot 0^2\right)}^{4}\Delta y \;\Big|\; \frac{d}{dt}\left(y-\frac{4}{3}\right) = \frac{d}{dt}\Delta y = \overbrace{\left(4-6\left(\frac{4}{3}\right)^2\right)}^{-4}\Delta y$$

Ha $y(0) = 0.5$, mennyi

$\lim_{x\to\infty} y(x) = \frac{4}{3}$ $\qquad$ $\lim_{x\to-\infty} y(x) = 0$

Vazold a DE megoldasgorbeit!



$\lim_{\ominus} \to 0$

4.b.

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_2(y_1-4) \\ (y_2+6)y_1 \end{pmatrix}$$

$$\left.\begin{aligned} y_2(y_1-4) &= 0 \\ (y_2+6)y_1 &= 0 \end{aligned}\right\} \Rightarrow \left[\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\vec{Z}_A} \text{ vagy } \underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}}_{\vec{Z}_B}\right]$$

Keresd meg a DE fixpontjat!

$$Jac = \begin{bmatrix} \partial_{y_1}[y_2(y_1-4)] & \partial_{y_2}[y_2(y_1-4)] \\ \partial_{y_1}[(y_2+6)y_1] & \partial_{y_2}[(y_2+6)y_1] \end{bmatrix} = \begin{bmatrix} y_2 & y_1-4 \\ y_2+6 & y_1 \end{bmatrix}$$

$$Jac(\vec{Z}_A) = \begin{bmatrix} 0 & -4 \\ +6 & 0 \end{bmatrix} \qquad Jac(\vec{Z}_B) = \begin{bmatrix} -6 & 0 \\ 0 & 4 \end{bmatrix}$$

Ird fel a fixpont koruli linearizalt kozelito DE-t!

$\vec{Z}_A$:

$$\frac{d}{dt}\begin{bmatrix} y_1-0 \\ y_2-0 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ +6 & 0 \end{bmatrix}\begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix}$$

$\vec{Z}_B$:

$$\frac{d}{dt}\begin{bmatrix} y_1-4 \\ y_2+6 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix}$$

Zh2. Diff.Egy., 2026.05.06. NEPTUN:

Nev: Alairas:

1.

a) Legyen $\dot{y}(t) = -2y(t) + f(t)$, $y(0) = 13$! $f(t) = 7 - t$

a1) Mennyi az $y(t)$ fuggveny $Y(s)$ Laplace transzformaltja?

$$Y(s) = \frac{1}{s-(-2)}\left(13 + \frac{7}{s} - \frac{1}{s^2}\right)$$

a2) Hogyan nez ki $Y(s)$ parcialis tort felbontasa? (A felmerulo egyutthatokat nem kell kiszamitani.)

$$Y(s) = \frac{A}{s+2} + \frac{B}{s} + \frac{C}{s^2}$$

a3) Hogyan nez ki $y(t)$?

$$y(t) = Ae^{-2t} + B + C \cdot t$$

b) Legyen $\ddot{y}(t) = -2\dot{y}(t) - 5y(t) + f(t)$, $y(0) = 13$! , $\dot{y}(0) = 8$, $f(t) = 7 - t$

b1) Mennyi az $y(t)$ fuggveny $Y(s)$ Laplace transzformaltja?

$$s^2 Y(s) - s \cdot 13 - 8 = -2[sY(s) - 13] - 5Y(s) = \frac{7}{s} - \frac{1}{s^2}$$

$$Y(s) = \frac{1}{s^2 + 2s + 5}\left(34 + 13s + \frac{7}{s} - \frac{1}{s^2}\right)$$

b2) Hogyan nez ki $Y(s)$ parcialis tort felbontasa? (A felmerulo egyutthatokat nem kell kiszamitani.)

$$Y(s) = \frac{A}{s-(-1+2i)} + \frac{B}{s-(-1-2i)} + \frac{C}{s^2} + \frac{D}{s}$$

b3) Hogyan nez ki $y(t)$?

$$y(t) = Ae^{(-1+2i)t} + Be^{(-1-2i)t} + Ct + D$$

2.

a1) Legyen $f(x) = sgn(x)$, ha $x \in (-\pi, \pi]$, es legyen $f(x)$ periodikus, 2π periodussal. Ha

$$f(x) = \sum_{n\in\mathbb{Z}} \hat{f}_n \frac{1}{\sqrt{2\pi}} e^{inx},$$

akkor mennyi $\hat{f}_n$?

ortonormált bázis: $e_n(x) = \frac{1}{\sqrt{2\pi}} e^{inx} = \vec{e}_n$, $n \in \mathbb{Z} = \dots -1, 0, 1, 2, \dots$

$$\hat{f}_n = (\vec{e}_n, f) = \int_{-\pi}^{\pi} \overline{\frac{1}{\sqrt{2\pi}} e^{inx}}\, f(x)\, dx = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} e^{-inx}\, sgn(x)\, dx =$$

$$= \frac{1}{\sqrt{2\pi}} \left[-1 \cdot \int_{-\pi}^{0} e^{-inx} dx + 1 \cdot \int_{0}^{\pi} e^{-inx} dx \right] = \frac{1}{\sqrt{2\pi}} \left[\left(-\frac{e^{-in\cdot 0}}{-in} \right) - \left(-\frac{e^{-in(-\pi)}}{-in} \right) + \left(\frac{e^{-in\pi}}{-in} \right) - \left(\frac{e^{-in\cdot 0}}{-in} \right) \right]$$

a2) Fejezd ki a kovetkezo differencialegyenletet

$$\partial_t \phi(t,x) = -\,\partial_x^2 \phi(t,x), \quad \phi(t,x) = \phi(t, x+2\pi), \; \phi(0,x) = f(x)$$

megoldasat $\hat{f}$ segitsegevel!

$$\varphi(x,t) = \sum_{n\in\mathbb{Z}} e^{+n^2 t} \vec{e}_n = \sum_{n\in\mathbb{Z}} e^{+n^2 t} \cdot \hat{f}_n \cdot \frac{1}{\sqrt{2\pi}} e^{inx}$$

a3) Magyarazd el, hogy mi a kulonbseg a megoldas helyessegenek tekinteteben, ha $t = 1$, vagy ha $t = -1$!

$t \to \infty$: $e^{+n^2 t} \hat{f}_n$ gyorsan növekszik, ahogy $t \to \infty$, $\varphi(x,t)$ divergens

$t \to -\infty$: $e^{+n^2 t} f_n$ gyorsan csökken ahogy $t \to -\infty$, $\varphi(x,t)$ konvergens lesz

b) Szamold ki a kovetkezo konvoluciokat!

i) $1 * 1$,

$$(1*1)(t) = \int_0^t 1 \cdot 1\, dt = t$$

ii) $t * t$,

$$(t*t)(t) = \int_0^t (t-\tau)\tau\, d\tau = \left[t\frac{\tau^2}{2} - \frac{\tau^3}{3} \right]_{\tau=0}^{t} = \frac{1}{6} t^3$$

ii) $t * \theta(t-2)$, ahol $\theta(t)$ az egysegugras fuggveny.

$$= \int_0^t (t-\tau)\theta(t-2)\, d\tau = \int_2^t t - \tau\, d\tau = \left[t\tau - \frac{\tau^2}{2} \right]_{\tau=2}^{t} = \left(t^2 - \frac{t^2}{2} \right) - \left(t \cdot 2 - \frac{2^2}{2} \right) =$$

$$= \frac{1}{2} t^2 - 2t + 2$$

3.

a1) Legyen a variacios problema Lagrange fuggvenye $L(t, x_1, \dot{x}_1, x_2, \dot{x}_2) = \dot{x}_1^2 + (1 + x_1x_2)\dot{x}_2^2 + \dot{x}_1\dot{x}_2$. Ird fel az $x(t)$ fuggvenyre vonatkozo Euler-Lagrange egyenletet!

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_1} - \frac{\partial L}{\partial x_1} = \frac{d}{dt}\left(2\dot{x}_1 + \dot{x}_2\right) - x_2\dot{x}_2^2 = 0 = \left(2\ddot{x}_1 + \ddot{x}_2\right) - x_2\dot{x}_2^2 = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_2} - \frac{\partial L}{\partial x_2} = \frac{d}{dt}\left(2\dot{x}_2(1+x_1x_2) + \dot{x}_1\right) - x_1\dot{x}_2 = 0$$

a2) Legyen a variacios problema Lagrange fuggvenye $L(t, x, \dot{x}) = x^{-4}\dot{x}^{-3} - t\cos(x)$. Ird fel az $x(t)$ fuggvenyre vonatkozo Euler-Lagrange egyenletet! Hanyadrendű differenciálegyenlet az eredmény?

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = \frac{d}{dt}\left(x^{-4}\cdot(-3)\,\dot{x}^{-4}\right) - \left(t\cdot[-\sin(x)]\right) = 0$$

Másodrendű : $\ddot{x} = \ldots$

b) Legyen

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} \cos(2t) \\ 2t \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$\cos(2t) = \frac{e^{-2it} + e^{2it}}{2}$$

Ird fel $\vec{y}(t)$ Laplace transzformaltjat! (A felmerulo matrixinvertalast nem kell elvegezni.)

$$\begin{bmatrix} s\cdot Y_1(s) - 6 \\ s\cdot Y_2(s) - 7 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} + \begin{bmatrix} \frac{1/2}{s-2i} + \frac{1/2}{s+2i} \\ 2\cdot\frac{1}{s^2} \end{bmatrix}$$

Tehát:

$$\vec{Y}(s) = \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} -3+s & -4 \\ -5 & -6+s \end{bmatrix}^{-1}\begin{bmatrix} 6 + \frac{1/2}{s-2i} + \frac{1/2}{s+2i} \\ 7 + 2\cdot\frac{1}{s^2} \end{bmatrix}$$

Haeviside egységugrás függvény

4.

a1) Keresd meg a $\dot{G}(t) = -2G(t) + \delta(t)$ DE retardalt megoldasat! Indokold a valaszodat!

$t<0$: $G(t)=0$ (retardáció)

$t\approx 0$: $G(t)\approx\Theta(t)$, $\dot{\Theta}(t)=\delta(t)$, $\Theta(0^-)=0 \longrightarrow \Theta(0^+)=1$, $G(0^+)=1$

$t>0$: Oldd meg: $\dot{y}=-2y$, $y(0)=G(0^+)=1 \longrightarrow y(t)=e^{-2t}$

Tehát

$$G(t)=\begin{cases}0, & t<0\\ e^{-2t}, & t>0\end{cases}$$

a2) Ird fel az $\dot{y}(t) = -2y(t) + f(t)$ DE megoldasat, ha $y(t) = f(t) = 0$ amikor $t << 0$!

$$y(t)=\int_{-\infty}^{t} e^{-2(t-\tau)} f(\tau)\,d\tau = \int_{-\infty}^{\infty} G(t-\tau)f(\tau)\,d\tau = (G\cdot f)(\tau)$$

a3) Ird fel az $\dot{y}(t) = -2\dot{y}(t) - \cancel{5y(t)} + f(t)$ DE megoldasat a $t > 0$ idopontokra, ha $y(0) = 13$! kissé "hibás" feladat!

$$y(t)=\int_{0}^{t} e^{-2(t-\tau)} f(\tau)\,d\tau + 13\underbrace{e^{-2t}}_{G(t)}$$

ez a $\dot{y}=-2y+f(t)$ megoldása

b1) Keresd meg a $\ddot{G}(t) = -2\dot{G}(t) - 5G(t) + \delta(t)$ DE retardalt megoldasat! Indokold a valaszodat!

$t<0$: $G(t)=0$ (retardáció)

$t\approx 0$: $G(t)\approx R(t)$, $\ddot{R}(t)=\delta(t)$, $(R(t)=0, \text{ ha } t<0) \longrightarrow R(t)=\begin{cases}0, & \text{ha } t<0\\ t, & \text{ha } t>0\end{cases}$

tehát $G(0^+)=0$, $\dot{G}(0^+)=1$

Oldd meg: $\ddot{y}=-2\dot{y}-5y$, $y(0)=G(0^+)=0$, $\dot{y}(0)=\dot{G}(0^+)=1$

$$\longrightarrow y(t)=\frac{1}{4}\left(-i e^{(-1+2i)t}+i e^{(-1-2i)t}\right)=\frac{1}{2}e^{-t}\sin(2t)$$

Tehát

$$G(t)=\begin{cases}0, & t<0\\ \frac{1}{2}e^{-t}\sin(2t), & t>0\end{cases}$$

b2) Ird fel az $\ddot{y}(t) = -2\dot{y}(t) - 5y(t) + f(t)$ DE megoldasat, ha $y(t) = f(t) = 0$ amikor $t << 0$!

$$y(t)=\int_{-\infty}^{\infty} G(t-\tau)f(\tau)\,d\tau = (G*f)(t) =$$

$$=\int_{-\infty}^{t} \frac{1}{2} e^{-(t-\tau)} \sin(2[t-\tau])\, f(\tau)\,d\tau$$