Diff. egy. Zh2, 24.05.15 Megoldás. (1-3)

1a $\dot{y} = 9y + t - 1,\ y(0) = 13$

a1 $sY(s) - 13 = 9Y(s) + \frac{1}{s^2} - \frac{1}{s} \longrightarrow Y(s) = \frac{1}{s-9}\left(\frac{1}{s^2} - \frac{1}{s} + 13\right)$ 2p

a2 $Y(s) = \frac{A}{s-9} + \frac{B}{s^2} + \frac{C}{s}$ 1p

a3 $y(t) = Ae^{9t} + Bt + C$ 1p

1b $\ddot{y} = -9y + 1 - t,\ y(0) = 13,\ \dot{y}(0) = 77$ 2p

b1 $s^2Y(s) - 13s - 77 = -9Y(s) - \frac{1}{s^2} + \frac{1}{s}$

$Y(s) = \frac{1}{s^2+9}\left(\frac{1}{s} - \frac{1}{s^2} + 13s + 77\right)$ 2p

b2 $Y(s) = \frac{A}{s+3i} + \frac{B}{s-3i} + \frac{C}{s^2} + \frac{D}{s}$ 2p

b3 $y(t) = Ae^{-3it} + Be^{3it} + Ct + D$

2a $\frac{d}{dt}\vec{y}(t) = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}\vec{y}(t),\ \vec{y}(0) = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

a1 $\begin{bmatrix} sY_1(s) - 5 \\ sY_2(s) - 6 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} \longrightarrow \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} s-4 & 3 \\ 2 & s-1 \end{bmatrix}^{-1}\begin{bmatrix} 5 \\ 6 \end{bmatrix}$ 4p

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b1 $\hat{f}_2 = (e_2(x), f(x)) = \int_0^{2\pi} \frac{e^{-2ix}}{\sqrt{2\pi}} \cdot f(x)\,dx = \int_0^1 \frac{e^{-2ix}}{\sqrt{2\pi}} \cdot 2\,dx + \int_1^{2\pi} \frac{e^{-2ix}}{\sqrt{2\pi}} \cdot 3\,dx =$ 3p

$= \frac{1}{\sqrt{2\pi}\cdot(-2i)}\left[2\cdot(e^{-2ix} - 1) + 3\cdot(1 - e^{-2ix})\right]$ 2p

b2 $\varphi(t,x) = \sum_{n\in\mathbb{Z}} e^{-3\cdot(in)^2\cdot t} \cdot \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}} = \sum_{n\in\mathbb{Z}} e^{3n^2t} \cdot \hat{f}_n \frac{e^{inx}}{\sqrt{2\pi}}$

b3 (i) $t=1$: $e^{+3n^2\cdot 1} \to \infty$, sor divergens, $\varphi(1,x)$ "értelmetlen". 1p

(ii) $t=-1$: $e^{+3n^2\cdot(-1)} \to 0$, sor konvergens, $\varphi(-1,x)$-et megadja a sor.

3a $L(\vec{x}, \dot{\vec{x}}) = (\dot{x}_1 - \dot{x}_2)^2 + \dot{x}_1 x_2 + x_1 x_2$

Euler-Lagrange: $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} - \frac{\partial L}{\partial x_i} = 0$

$i=1$: $\frac{d}{dt}\left(2(\dot{x}_1 - \dot{x}_2) + x_2\right) - x_2 = 0 \longrightarrow 2\ddot{x}_1 - 2\ddot{x}_2 + \dot{x}_2 - x_2 = 0$ 4p

$i=2$: $\frac{d}{dt}\left(2(\dot{x}_1 - \dot{x}_2)\cdot(-1)\right) - (\dot{x}_1 + x_1) = 0 \longrightarrow -2\ddot{x}_1 + 2\ddot{x}_2 - (\dot{x}_1 + x_1) = 0$

3b $\dot{y} = 3y + \Theta(t),\ y(-77) = 0$

b1 (i) $y(t) = 0$, ha $t < 0$.

(ii) $t > 0$: $y(0) = 0$, $\dot{y} = 3y + 1 \longrightarrow y(t) = -\frac{1}{3} + \frac{1}{3}e^{3t}$ $\Big\} \longrightarrow y(t) = \begin{cases} 0, & \text{ha } t < 0 \\ \frac{1}{3}(e^{3t} - 1), & \text{ha } t > 0 \end{cases}$ 4p

b2 Mivel $\dot{\Theta}(t) = \delta(t)$, így az $\dot{y}(t) = 3y(t) + \delta(t)$ megoldása

$y(t) = G(t) = \frac{d}{dt}\left[\begin{cases} 0, & \text{ha } t < 0 \\ \frac{1}{3}(e^{3t} - 1), & \text{ha } t > 0 \end{cases}\right] = \begin{cases} 0, & \text{ha } t < 0 \\ e^{3t}, & \text{ha } t > 0 \end{cases}$ 2p

Diff. egy. Zh2. 24.05.15 Megoldás. 4

4 (a1) $\dot G(t) = 9G(t) + \delta(t)$. Mi a retardált megoldás?

(i) $t<0$: $G(t)=0$

(ii) $t\approx 0$: $G(t)\approx \Theta(t)$, mivel $\dot\Theta(t)=\delta(t)$

$\Theta(0^-)=0=G(0^-)$, $\Theta(0^+)=1=G(0^+)$

(iii) $t>0$: Oldd meg: $\dot y(t)=9y(t)$, $y(0)=G(0^+)=1 \longrightarrow y(t)=e^{9t}$

tehát $G(t)=\begin{cases}0, & t<0\\ e^{9t}, & t>0\end{cases}$ 3p

(a2) $y(t)=(G*f)(t)=\int_{-\infty}^{\infty} G(t-\tau)f(\tau)\,d\tau=\int_{-\infty}^{t} e^{9(t-\tau)}f(\tau)\,d\tau$ 1p

(a3) $y(t)=13G(t)+\int_0^t e^{9(t-\tau)}f(\tau)\,d\tau$ 1p

(b1) $\ddot G=9G(t)+\delta(t)$. Mi a retardált megoldás?

(i) $t<0$: $G(t)=0$

(ii) $t\approx 0$: $G(t)\approx R(t)=\begin{cases}0, & t<0\\ t & t>0\end{cases}$

$G(0^-)=R(0^-)=0$, $G(0^+)=R(0^+)=0$

$\dot G(0^-)=\dot R(0^-)=0$, $\dot G(0^+)=\dot R(0^+)=1$

(iii) $t>0$: Oldd meg: $\ddot y=9y$, $y(0)=G(0^+)=0$, $\dot y(0)=\dot G(0^+)=1$,

vagyis $y(t)=\frac{1}{2\cdot 3}\left(e^{3t}-e^{-3t}\right)$

tehát $G(t)=\begin{cases}0, & t<0\\ \frac{1}{6}\left(e^{3t}-e^{-3t}\right), & t>0\end{cases}$ 4p

(b2) $y(t)=(G*f)(t)=\int_{-\infty}^{\infty} G(t-\tau)f(\tau)\,d\tau=\int_{-\infty}^{t}\frac{1}{6}\left(e^{3(t-\tau)}-e^{-3(t-\tau)}\right)f(\tau)\,d\tau$ 1p