$m = 3$ $\quad k = 1$

köbös elsőrendű spline.

$f:[a,b]\subset \mathbb{R}\to\mathbb{R}$

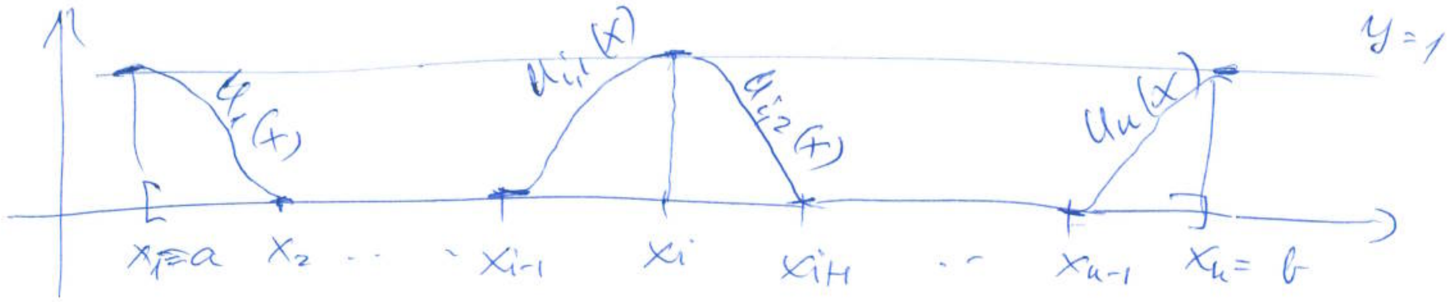
$x_1=a<x_2 \dots < x_n = b$

$f(x_1), f(x_2), \dots, f(x_n)$

$f'(x_1), f'(x_2), \dots, f'(x_n)$

$$S(x)=\sum_{i=1}^{n} u_i(x)\cdot f(x_i) + \sum_{j=1}^{n} v_j(x)\cdot f'(x_j)$$

Szakaszonként legfeljebb 3-adfokú polinom

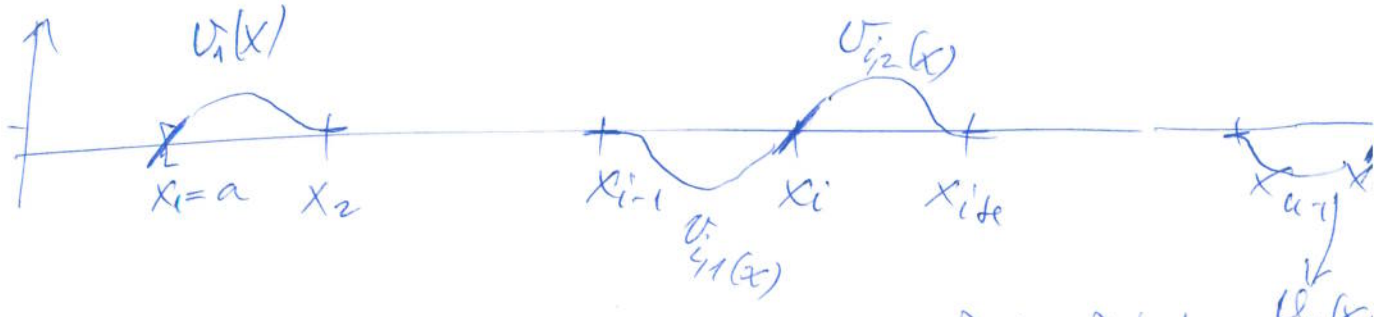


$$u_i(x)=\begin{cases} u_{i,1}(x) & \text{ha } x\in[x_{i-1},x_i] \\ u_{i,2}(x) & \text{ha } x\in[x_i,x_{i+1}]\end{cases}$$

$u_i: [x_{i-1},x_{i+1}]\subset[a,b]\to\mathbb{R}$

$\Downarrow$ szakaszonként legfeljebb 3-adfokú fgv.

$$u_i(x_j)=\begin{cases}1 & \text{ha } j=i\\ 0 & \text{egyébként}\end{cases} \qquad u_i'(x_j)=0 \quad j=i-1,i,i+1$$



$v_i : [x_{i-1}, x_{i+1}] \subset [a,b] \to \mathbb{R}$ szakaszonként legf. 3-adfokú pol.

$$v_i(x) = \begin{cases} v_{i,1}(x) & \text{ha } x \in [x_{i-1}, x_i] \\ v_{i,2}(x) & \text{ha } x \in [x_i, x_{i+1}] \end{cases}$$

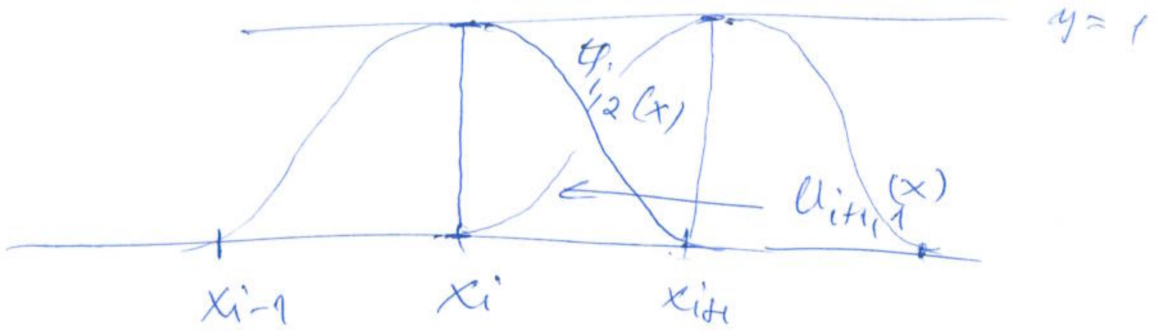
$v_i(x_j) = 0 \quad j = i-1, i, i+1 \qquad v_i'(x_j) = \begin{cases} 1 & \text{ha } j = i \\ 0 & \text{egyébként} \end{cases}$

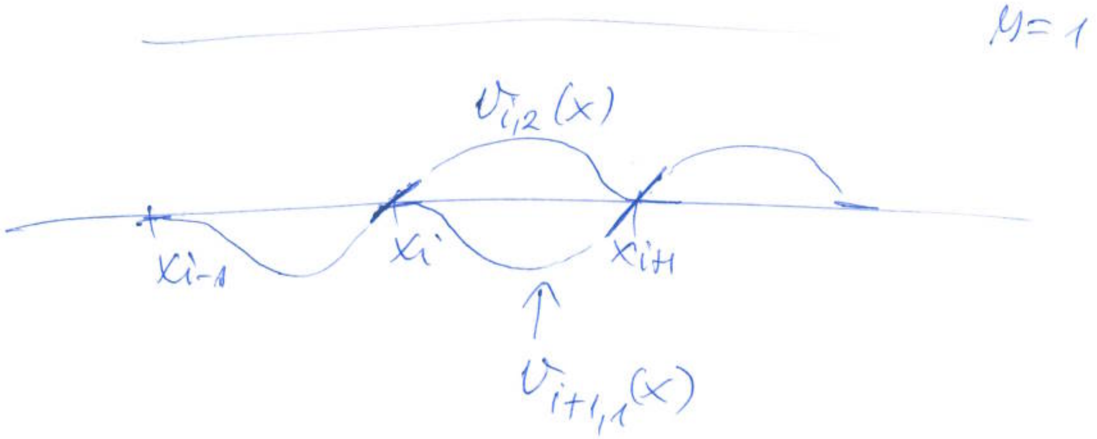
$$s(x) = u_1(x)\cdot f(x_1) + u_2(x)\cdot f(x_2) + \dots + u_n(x)\cdot f(x_n) + v_1(x)\cdot f'(x_1) + v_2(x)\cdot f'(x_2) + \dots + v_n(x)\cdot f'(x_n)$$

$$s'(x) = u_1'(x)\cdot f(x_1) + u_2'(x) f(x_2) + \dots + u_n'(x)\cdot f(x_n) + v_1'(x)\cdot f'(x_1) + v_2'(x)\cdot f'(x_2) + \dots + v_n'(x)\cdot f'(x_n)$$

$$S(x) = \begin{cases} s_1(x) & \text{ha } x \in [x_1, x_2] \\ s_2(x) & \text{ha } x \in [x_2, x_3] \\ \vdots & \\ s_i(x) & \text{ha } x \in [x_i, x_{i+1}] \\ \vdots & \\ s_{n-1}(x) & \text{ha } x \in [x_{n-1}, x_n] \end{cases}$$

$s_i : [x_i, x_{i+1}] \subset [a,b] \to \mathbb{R}$ legf. 3-adfokú pol.





$$s_i(x) = u_{i,2}(x) \cdot f(x_i) + u_{i+1,1}(x) \cdot f(x_{i+1}) +$$
$$+ v_{i,2}(x) \cdot f'(x_i) + v_{i+1,1}(x) \cdot f'(x_{i+1})$$

$(1 \le i \le n-1)$

Interp. köbös 1. rendű splinenal $F2_1$

Feladat: Adott az

x	-2	$-1,3$	0	1	2	$2,1$	3
$f(x)$	1	-10	1	0	1	0	2
$f'(x)$	1	10	-2	0	4	3	4

fv. táblázat. Határozza meg az $f(X)$, valamint $f'(X)$ ~~ért~~ közelítő értékét $x^* = 0,5$ pontban, ~~köbös 1. rendű spline bázisfvének~~
Oldja meg a feladatot köbös 1. rendű ~~splinefv. bázisfv.-nyének felhasználásával előállításával!~~ spline alkalmazásával!

$\boxed{\text{Megoldás}}$

$\boxed{F2_2}$

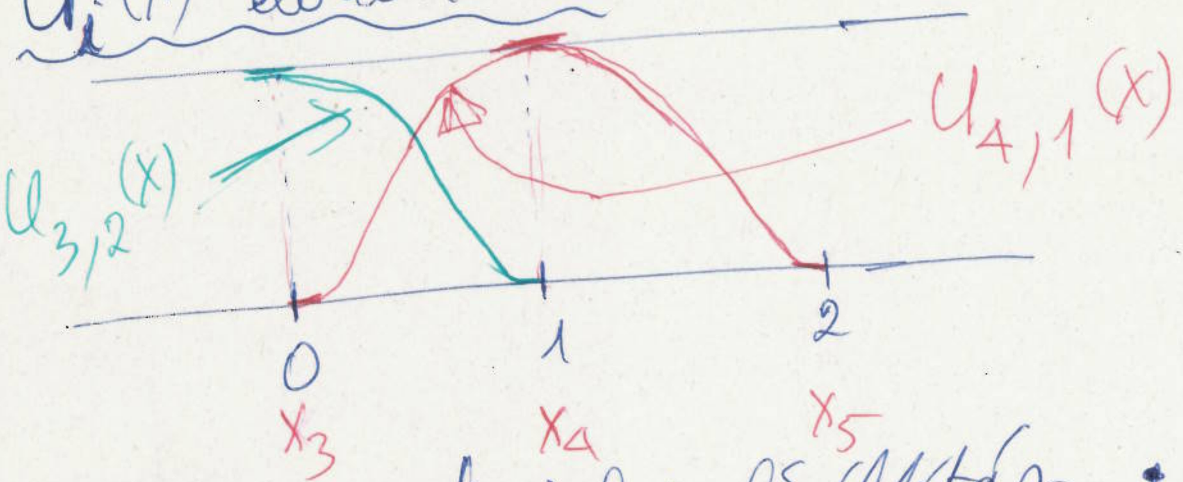
(a) Először eldöntjük, hogy melyik alapponthoz van legközelebb az $x^* = 0{,}5$.

$0{,}5 \in [0, 1]$

~~$|0{,}5 - (-1{,}3)| = 1{,}8$~~

~~$|0{,}5 - 2| = 1{,}5$~~

$U_i(x)$ előállítása:



$U_{3,2}(x)$ bázisfv előállítása:

$$U_{3,2}(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$U'_{3,2}(x) = \qquad a_2 + 2a_3 x + 3a_4 x^2$$

$= 1$

$a_1 \qquad = 0$

$a_1 + a_2 + a_3 + a_4$

$= 0$

a_2

$a_2 + 2a_3 + 3a_4 = 0$

~~F2~~ $\boxed{F2_3}$

$$a = \begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix} \Longrightarrow \cancel{u_{3,2}(x) =}$$

$$\boxed{u_{3,2}(x) = 1 - 3x^2 + 2x^3}$$

$u_{4,1}(x)$ előállítása

$$u_{4,1}(x) = b_1 + b_2 x + b_3 x^2 + b_4 x^3$$

$$u'_{4,1}(x) = b_2 + 2b_3 x + 3b_4 x^2$$

$$\begin{aligned}
b_1 &= 0 \\
b_1 + b_2 + b_3 + b_4 &= 1 \\
b_2 &= 0 \\
b_2 + 2b_3 + 3b_4 &= 0
\end{aligned}$$

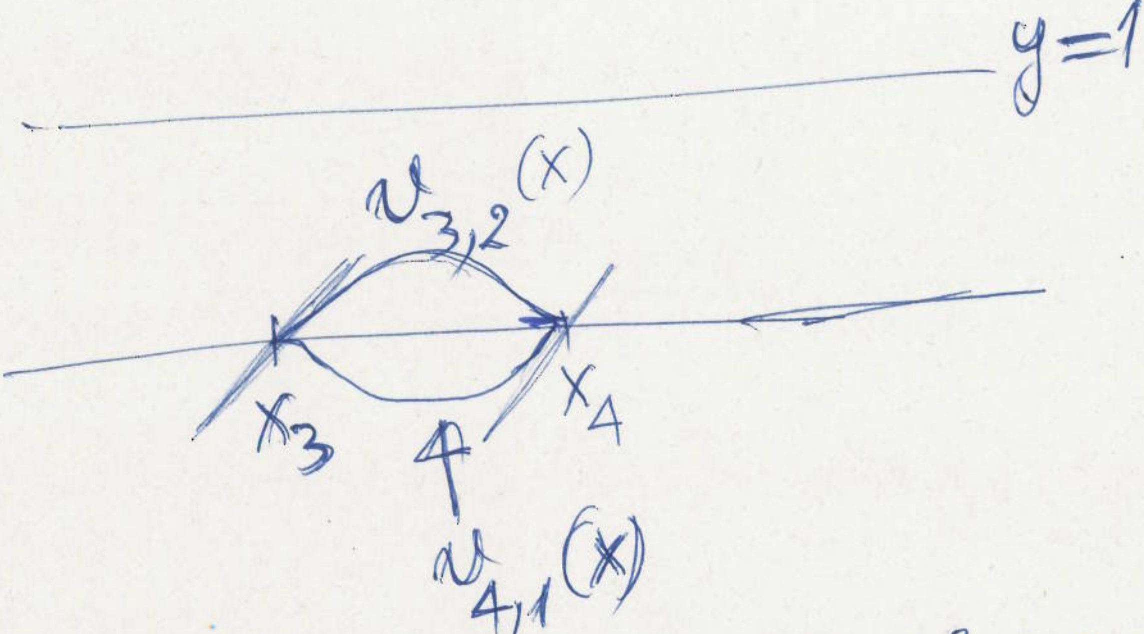
$$b = [0, 0, 3, -2]^T$$

$$u_{4,1}(x) = 3x^2 - 2x^3$$

$$\cancel{S_3(x) = l_{3,2}(x) f(x_3) + l_{4,1}(x) \cdot f(x_4)}$$

$$\cancel{=}$$

F2$_A$



$$v^*_{3,2}(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3$$

$$v'_{3,2}(x) = \quad c_2 + 2c_3 x + 3c_4 x^2$$

$$\begin{aligned} c_1 \quad &= 0 \\ c_1 + c_2 + c_3 + c_4 &= 0 \\ c_2 \quad &= 1 \\ c_2 + 2c_3 + 3c_4 &= 0 \end{aligned}$$

$$c = [0, 1, -2, 1]$$

$$v_{3,2}(x) = x - 2x^2 + x^3$$

$v_{4,1}(x)$ előállítása ;

$\boxed{F2_5}$

$$v_{4,1}(x) = d_1 + d_2 x + d_2 x^2 + d_3 x^3$$

$$v'_{4,1}(x) = d_2 + 2d_2 x + 3d_3 x^2$$

$$\cancel{= a} = 0$$

$$d_1 = 0$$

$$d_1 + d_2 + d_3 + d_4 = 0$$

$$d_2 = 1$$

$$d_2 + 2d_3 + 3d_4$$

$$d = [0, 0, -1, 1]^T$$

$$v_{4,1}(x) = -x^2 + x^3$$

$$S_3(x) = u_{3,2}(x) \cdot f(x_3) + u_{4,1}(x) \cdot f(x_4) + v_{3,2}(x) \cdot f'(x_3) + v_{4,1}(x) \cdot f'(x_4) =$$

$$= 1 - 3x^2 + 2x^3 + \cancel{2}(x - 2x^2 + x^3) \cdot (-2)$$

$$= \cancel{-5}$$

$$= 1 - 3x^2 + \cancel{2x^3} - 2x + 4x^2 - \cancel{2x^3}$$

$$= \cancel{1 + x^2}\ 1 - 2x + x^2$$

$$S_3'(x) = -2 + 2x$$

$$f(0{,}5) \approx S_3(0{,}5) = \underbrace{1 - 0{,}5 \cdot 2}_{0} + 0{,}5^2$$

$$f(0{,}5) \approx 0{,}2500$$

$$f'(0{,}5) \approx S_3'(0{,}5) = -2 + 2 \cdot 0{,}5 =$$
$$= -1$$

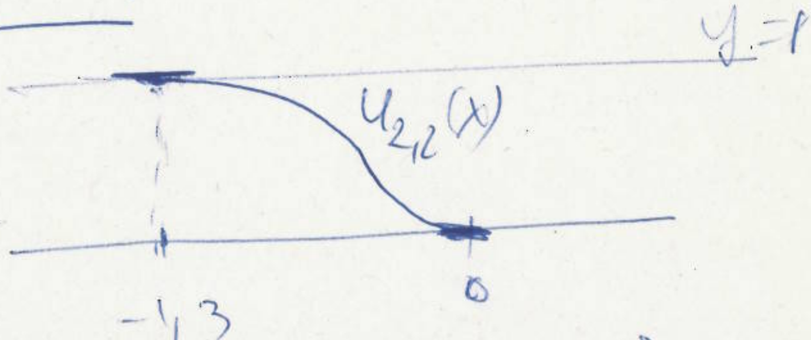
$$f'(0{,}5) \approx -1$$

Feladat : ~~Adott táblás~~

Adott futáblázat esetén állítsa elő az $u'_{2,2}(x)$ és $v'_{3,1}(x)$ bázis függvényeket!

$u_{2,2}(x)$ előállítása :

$$\underset{x_2}{[-1,3} \; ; \; \underset{x_3}{0]}$$



$$\cancel{u_{2,2}(x)} = u'_{2,2}(x) = a_2 + 2a_3 x + 3a_4 x^2$$

$$u_{2,2}(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$a_1 - 1,3 a_2 + 1,69 a_3 - 2,197 a_4 = 1$$

$$= 0$$

$$a_1 \qquad = 0$$

$$a_2 - 2,6 a_3 + 5,07 a_4 = 0$$

$$a_2 \qquad = 0$$

$$a = \begin{bmatrix} 0 \\ 0 \\ 1,7751 \\ 0,9103 \end{bmatrix}$$

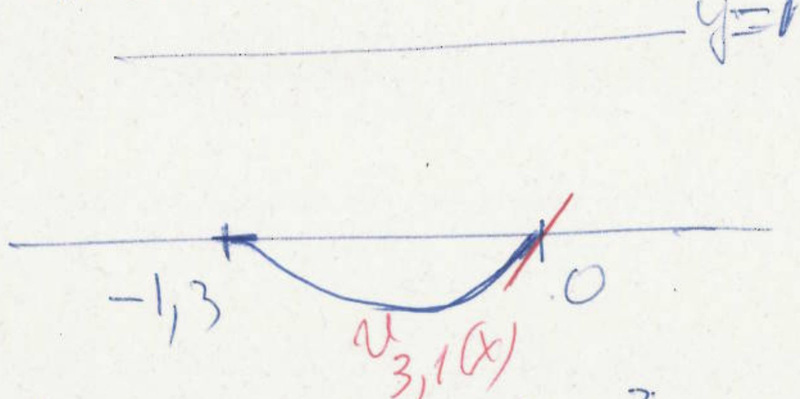
$$u_{2,2}(x) = 1,7751 x^2 + 0,9103 x^3$$

$$\boxed{u'_{2,2}(x) = 3,5502 x + 2,7309 x^2}$$

$v_{3,1}(x)$ előállítása:

$[-1,3\ ;\ 0]$ interv.

$\quad x_2 \qquad x_3$



$$v_{3,1}(x) = b_1 + b_2 x + b_3 x^2 + b_4 x^3$$

$$v'_{3,1}(x) = b_2 + 2b_3 x + 3b_4 x^2$$

$$\begin{aligned}
b_1 - 1,3\,b_2 + 1,69\,b_3 - 2,197\,b_4 &= 0 \\
b_1 \qquad\qquad\qquad\qquad\qquad &= 0 \\
b_2 - 2,6\,b_3 + 5,07\,b_4 &= 0 \\
b_2 \qquad\qquad\qquad\qquad &= 1
\end{aligned}$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1,5385 \\ 0,5917 \end{bmatrix}$$

$$v_{3,1}(x) = x + 1,5385x^2 + 0,5917x^3$$

$$\boxed{v'_{3,1}(x) = 1 + 3,0770x + 1,7751\,x^2}$$