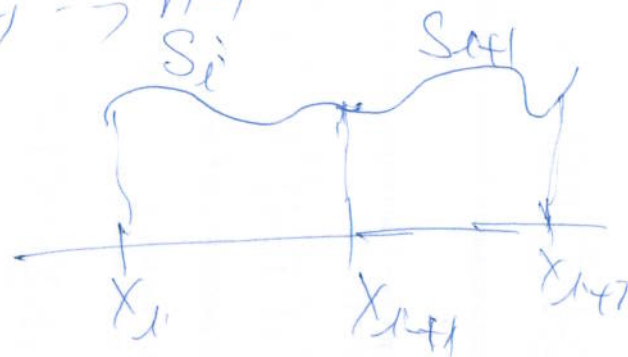


Természetes Spline-ek def és egyenletek

$$\left. \begin{aligned} S_i(x_i) &= f(x_i) \\ S_i(x_{i+1}) &= f(x_{i+1}) \end{aligned} \right] i=1, 2, \dots, n-1$$



~~$$S_{i+1}(x_{i+1}) = f(x_{i+1})$$~~

$$\left. \begin{aligned} S'_i(x_{i+1}) &= S'_{i+1}(x_{i+1}) \\ S''_i(x_{i+1}) &= S''_{i+1}(x_{i+1}) \end{aligned} \right] i=1, \dots, n-2$$

$$S''_1(x_1) = 0$$

$$S''_{n-1}(x_n) = 0$$

def: A köbös másodrendű spline függvényt, melynek 2. deriváltja zérus az adott $[a, b]$ intervallum végpontjaiban, természetes spline-nak ~~ne~~ nevezünk.

Természetes spline

Tspline

Feladat: Az

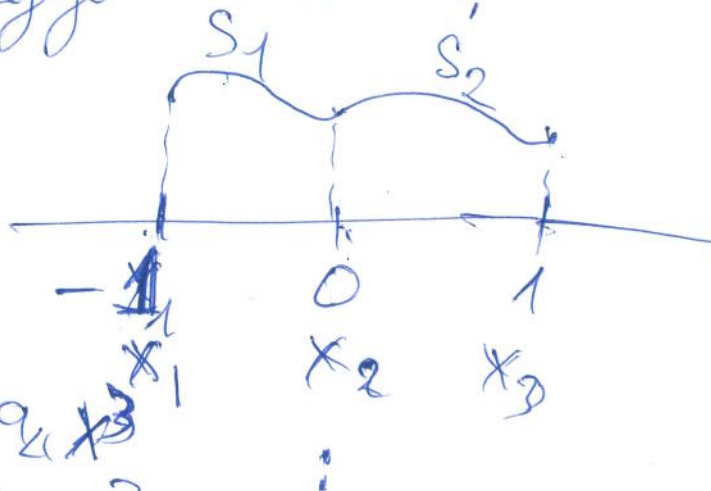
x	-1	0	1
$f(x)$	$1/2$	0	1

 fv táblázat

esetén ~~ho~~ adjuk ~~fel~~ a természetes spline fv-t def lb egyenleteket!

Megold

$$n = 3$$



$$S_1(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$S_2(x) = b_1 + b_2 x + b_3 x^2 + b_4 x^3$$

$$S'_1(x) = a_2 + 2a_3 x + 3a_4 x^2$$

$$S'_2(x) = b_2 + 2b_3 x + 3b_4 x^2$$

$$S''_1(x) = 2a_3 + 6a_4 x$$

$$S''_2(x) = 2b_3 + 6b_4 x$$

~~(n=3)~~

$$2n-2=4 \text{ db}$$

$$S_1(-1) = f(-1)$$

$$S_1(0) = f(0)$$

$$S_2(0) = f(0)$$

$$S_2(1) = f(1)$$

$$2n-4=2 \text{ db}$$

$$n=3 \Rightarrow \cancel{2n-4=2}$$

$$x=1, \quad n-2=1$$

$$S'_1(0) = S'_2(0)$$

$$S''_1(0) = S''_2(0)$$

$$S''_1(-1) = 0$$

$$S''_2(1) = 0$$

Tspl₂

$$a_1 - a_2 + a_3 - a_4 = 0,5$$

$$a_1 = 0$$

$$b_1 = 0$$

$$b_1 + b_2 + b_3 + \cancel{b_4} = 1$$

$$a_2 = b_2$$

$$2a_3 = 2b_3$$

$$2a_3 - 6a_4 = 0 \rightarrow a_4 = \frac{1}{3}a_3$$

$$2b_3 + 6b_4 = 0 \rightarrow b_4 = -\frac{1}{3}b_3 = -\frac{1}{3}a_3$$

$$-a_2 + a_3 - \frac{1}{3}a_3 = 0,5$$

$$-a_2 + \frac{2}{3}a_3 = 0,5$$

$$a_2 + a_3 - \frac{1}{3}a_3 = 1$$

$$a_2 + \frac{2}{3}a_3 = 1$$

$$-a_2 + \frac{2}{3}a_3 = 0,5$$

$$a_2 + \frac{2}{3}a_3 = 1$$

$$\frac{8}{3}a_3 = 1,5$$

$$a_3 = \frac{1,5}{8} = 0,1875$$

$$a_2 = 1 - \frac{2}{3} \cdot 0,5625 = 0,6250$$

$$a_4 = \frac{1}{3} \cdot 0,5625 = 0,1875$$

$$a = \begin{bmatrix} 0 \\ 0,6250 \\ 0,5625 \\ 0,1875 \end{bmatrix}$$

$$b_1 = 0$$

$$b_2 = 0,6250$$

$$b_3 = 0,5625$$

$$b_4 = -\frac{1}{3} \cdot 0,5625 = -0,1875$$